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Political motivations and electoral competition: Equilibrium analysis and experimental evidence

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A B S T R A C T

We study both theoretically and experimentally the set of Nash equilibria of a classical one-dimensional election game with two candidates. These candidates are interested in power and ideology, but their weights on these two motives are not necessarily identical. Apart from obtaining the well known median voter result and the two-sided policy differentiation outcome, the paper uncovers the existence of two new equilibrium configurations, called 'one-sided' and 'probabilistic' policy differentiation, respectively. Our analysis shows how these equilibrium configurations depend on the relative interests in power (resp., ideology) and the uncertainty about voters' preferences. The theoretical predictions are supported by the data collected from a laboratory experiment, as we observe convergence to the Nash equilibrium values at the aggregate as well as at the individual levels in all treatments, and the comparative statics effects across treatments are as predicted by the theory.

1. Introduction

The *spatial* theory of electoral competition begins with the seminal contributions of Hotelling (1929) and Downs (1957). The basic model considers a majority rule election where two political candidates compete for office by simultaneously and independently proposing a platform from a unidimensional policy space (e.g., an income tax rate). As is well known in the literature, the equilibrium predictions of this model depend crucially on candidates' motivations for running for office. In this paper, we study the implications of the so-called *mixed motivations* hypothesis, according to which candidates are concerned not only about winning the election and being in power, but also about the ideological position of the policy implemented afterwards.[1]

Although this assumption is thoroughly familiar in its symmetric version, that is, when both candidates assign the *same* relative weight to their policy preference versus their desire to win office, what happens in the asymmetric scenario remains an open question. As we argue below, this case is not only interesting from a theoretical point of view, but also empirically relevant. Here, we offer a full characterization of the set of Nash equilibria for both cases, the symmetric and the asymmetric one, uncovering interesting (and sometimes counter-intuitive) equilibrium predictions that had not been identified yet in the literature. In addition, we conduct a laboratory experiment to assess whether the predictions of the model possess

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[1] This was first suggested by Calvert (1985), and it has been recently used in a number of papers, including Ball (1999), Groseclose (2001), Aragones and Palfrey (2005), Duggan and Fey (2005), Saporiti (2008), Callander (2008), Bernhardt et al. (2009), and Saporiti (forthcoming).

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any empirical relevance, studying in a rich set of treatments not only convergence of subjects' behavior to the theoretical predictions, but also learning and a number of comparative statics effects resulting from changing the interests in power (resp., ideology) and the uncertainty about voters' preferences.[2]

An important motivation for this research is that conceptually the mixed motivations hypothesis is more realistic than the traditional hypotheses of candidates' motivations, according to which candidates care in the *same* way and *only* about either winning power or policy. In a democracy, the mixed motivations probably emerge naturally from the fact that candidates are representatives of complex political organizations. To elaborate, in real world politics to reach the stage of being in competition for public office, citizens must first be nominated within the political parties; and for that to happen they need the support of regular party members, who are arguably much more concerned about the policies implemented after the election than about the actual winner of the contest. Thus, although politicians as other professionals might be more interested in their careers and, therefore, in winning the elections, it seems reasonable to expect that policy considerations will also enter into the candidate's payoff function with some weight.[3]

These weights need not be the same for all candidates. They could depend for instance on the features of the political organization that the candidate represents, such as the number of regular members, the level of activism within the organization, the internal process to nominate candidates, etc. The value of winning the election might also vary depending on whether the party of the candidate is the incumbent in office or a challenger. Thus, there seem to be ample reasons why one might expect asymmetric electoral motivations to be quite general. Some evidence seems to suggest that they may have some empirical relevance as well. An interesting case in this regard is the Radical and the Peronist Parties in Argentina. These two parties are the main political actors of the country. The Radical Party has been ever since its creation an ideological party, whereas Peronism has been a "movement", as Perón used to call it, basically motivated by being in power. Another case is the Labour and the Conservative Party in the UK election of 1997, in which both located on the center-right of the political spectrum.

A second motivation for this work is that from a theoretical point of view, the mixed motivations hypothesis has been shown to have nontrivial implications for the predictive power of the theory of electoral competition. In effect, Ball (1999) pointed out that, due to the discontinuities of the payoff functions, the electoral contest with hybrid motives does not always possess a Nash equilibrium in pure strategies. Moreover, it has been shown that the source of this instability can be attributed entirely to the asymmetric nature of the political goals (Saporiti, 2008). Yet, in spite of this, the analysis of the full set of Nash equilibria under this assumption remains an open question. Clearly, filling out this gap seems quite important, because elections play a central role in many economic models, particularly in models of political economy and public finance.

The main results of this paper can be summarized as follows. On the one hand, consistent with the theory already known, our equilibrium analysis shows that when the value of being in office is the *same* for the two candidates, both players announce either (i) a platform located on the estimated median ideal point (policy convergence) if the electoral uncertainty is *low* compared with the interest in office, or (ii) a platform located on their own ideological side (two-sided policy differentiation) if the uncertainty is *high*.[4]

On the other hand, when candidates have *asymmetric* motivations, the median voter result still dominates for low levels of uncertainty. However, as the uncertainty increases, i.e., as the length of the interval over which the median is distributed increases, first an equilibrium in pure strategies fails to exist. In that region, both candidates randomize optimally on one side of the median to avoid being copied and undercut by their rival (probabilistic differentiation). Second, outside that region, a pure strategy equilibrium is reestablished, but the two candidates assign all of the probability mass to a different platform. These policies are located initially on the same ideological side (one-sided policy differentiation), and then, as uncertainty further increases, on each candidate's political ground (two-sided differentiation).

The data collected from the experiment are largely supportive of these theoretical predictions. First, we find in all treatments that the median behavior of the left- and the right-wing subjects converge to the Nash equilibrium values. This happens even in the probabilistic differentiation treatment, with a unique mixed strategy equilibrium (MSE). In that treatment, we observe not only that subjects' choices approximate the bounds and the median of the MSE support, but also that the empirical cumulative distributions are close to the theoretical ones, with the cumulative distribution of the left-wing players first-order stochastically dominating the distribution of the right-wing players.

Second, in the symmetric motivations treatments, we note that the confidence intervals we construct around the medians shrink over time as well, indicating behavior that is consistent with the Nash equilibrium not only at the aggregate level but also at the individual level. In the asymmetric treatments, with one-sided policy differentiation in either pure or mixed strategies, some noise in the individual choices persists even after sixty rounds (elections) of play. This is consistent with equilibrium behavior in the treatment with a mixed strategy equilibrium, but not with equilibrium behavior in the treatment

[2] The use of experimental methods as opposite to field methods seems preferable to test the theory because the former allow for a level of control that cannot be achieved with the latter given the large number of confounds that influence the behaviors of interest.

[3] Morton (1993) reports on subjects in a laboratory experiment placing a weight of approximately 32% on winning the election, and 68% on the expected utility from the implemented platforms.

[4] In this paper, candidates' preferred policies are assumed to be distributed on either side of the median ideal point, so that the ideology of one candidate lies on the left and the other on the right.

with a pure strategy equilibrium, where, although the deviations diminish somewhat over time, they tend to be skewed to the center of the policy space.

Third, we find that subjects' learning takes place mainly within the first ten periods (elections), and that most of that learning does not vanish as subjects interchange their roles between candidates of different ideologies. Finally, in line with the theory, the comparative statics analysis across treatments confirm the theoretical predictions that (i) an increase in the electoral uncertainty leads to an increase in policy divergence; (ii) policy convergence is reestablished as both candidates become more office-motivated; (iii) the extent of the empirical differentiation on either side of the median is independent of candidates' ideologies; and (iv) an asymmetric increase in candidates' interests in power leads to policy divergence on one side of the median.

The rest of the paper is organized as follows. Section 2 discusses the related literature. Section 3 outlines the model of electoral competition. Section 4 derives the theoretical results, which are proved in Appendix A. Section 5 presents the experimental design, and Section 6 discusses the experimental evidence. Final remarks are made in Section 7.

2. Related literature

The literature on electoral competition is vast. We focus here only on those papers that are most relevant for our work. For a more comprehensive review, the suggested references are Osborne (1995), Roemer (2001) and Austen-Smith and Banks (2005).

On the theoretical front, this paper relates to two segments of the existing literature that deal with, respectively, elections with office and policy motivations, and elections with advantaged candidates. In the first segment, the first article to consider mixed motivations is Calvert (1985), though it does not go beyond offering a continuity result according to which *small* departures from office motivation and certainty lead to only *small* departures from policy convergence. Ball (1999) and Bernhardt et al. (2009) further examine the implication of this assumption. The first paper focuses on equilibrium existence, whereas the latter analyzes mainly the implication of the symmetric mixed motivations on voters' welfare. Differently from these contributions, our work focuses on a full equilibrium characterization and on the empirical validity of our theoretical findings, rather than on existence or welfare considerations.

The existence of Nash equilibrium in electoral competition with mixed motivations is also the focus of Saporiti (2008). That article shows that, in contrast with the usual causes behind the nonexistence of equilibria in the traditional models of electoral competition, essentially, the multi-dimensionality of the policy space and the heterogeneity of voters' preferences, the lack of pure strategy equilibria in one-dimensional contests with mixed motives and electoral uncertainty is due to the heterogeneity or asymmetry of interests of the political candidates. Saporiti (2008) proves the existence and uniqueness of a pure strategy equilibrium when candidates possess mixed but symmetric motivations; and it shows that the mixed extension of the hybrid election game satisfies Reny's (1999) better-reply security and, consequently, that a Nash equilibrium exists regardless of the nature of candidates' aspirations. The paper however is totally silent about the nature of the equilibrium policies. Our analysis here extends Saporiti (2008) not only by providing a complete characterization of the equilibrium policies, but also by testing the main empirical restrictions in the lab.

In addition to the articles mentioned above, there is a large number of papers that adopt the mixed motivations assumption and simultaneously alter other features of the basic framework. To mention a few, Aragones and Palfrey (2005) study a general incomplete information model of candidate quality allowing for heterogeneity in valence, ideology, and motivations. Callander (2008) considers a model with either policy or office motivated candidates, private information about candidates' types, and partial commitment at the electoral stage. In a more significant departure, Roemer (1999) analyzes a model where parties represent different constituencies, or economic classes, with well defined policy preferences. Parties are also integrated by opportunistic individuals who desire only to win office. Roemer assumes that each party must reach inner-party unanimity to formulate a proposal, and he proves the existence of a so-called *party unanimity Nash equilibrium*. Finally, Snyder and Ting (2002) model political parties as informative brands to voters, in a setup where candidates are driven by achieving office and, if elected, policy, and they need parties to credibly signal their true policy preferences.

Insofar as a relatively more office-motivated candidate has in equilibrium a higher probability of winning the election, this paper is also connected with the literature on elections with advantaged candidates. Starting with Ansolabehere and Snyder (2000), Groseclose (2001), and Aragones and Palfrey (2002), there is now a sizeable literature that analyzes candidates' behavior in the presence of valence advantage. This includes the previous articles plus several recent papers, including Kartik and McAfee (2007), Ashworth and Bueno de Mesquita (2009), Hummel (2010), and Iaryczower and Mattozzi (2013), among others.

An interesting feature in some of these works is that, as happens in our case, equilibria in mixed strategies emerge because the advantaged candidate is willing to copy the position of the disadvantaged one, forcing the latter to randomize in order to not be predictable. Notice however that there is a slightly different flavor in our framework from what we have in the valence models. In this paper, it is not the case that voters have a preference for a certain candidate, whom the other tries to mimic, but rather the electoral advantage emerges endogenously because candidates have different motivations for power and they react differently to the uncertainty about voters' preferences.

On the empirical front, our paper adds to the experimental literature that analyzes elections and candidate competition.[5] In that literature, there is, first, a number of early laboratory tests, surveyed by McKelvey and Ordeshook (1990), that examine the hypothesis of policy convergence to the median ideal point in the Downsian framework with purely office-motivated candidates. This early research has been later complemented by Morton (1993), who conducts a laboratory experiment to assess the hypothesis that platforms diverge when candidates are purely ideological and there is uncertainty about voters' preferences. More recently, Aragones and Palfrey (2004) report experimental results about the effects of valence asymmetries on the location of the equilibrium policies. None of the existing papers however have analyzed yet in the lab the case of mixed and, especially, asymmetric electoral motivations, which is precisely our contribution here.

Finally, to the extent that some of the equilibria in the asymmetric motivation case are in mixed strategies, this paper also complements the existing laboratory and field studies that look at how people behave in games with mixed strategy equilibria. Camerer (2003) provides an overview of the most relevant papers, with the main message being that although aggregate behavior is usually close to the equilibrium predictions, there are still significant deviations from them.[6] In our experiment, subjects entered a pure strategy in each period. Thus, we are agnostic about to what extent they actually mixed their strategies. However, the data shows that our subject pool make choices that closely approximate the mixed strategy predictions, 'as if' they were changing their play in order to avoid being predictable and exploited by their opponents.

3. The model

Two candidates, indexed by $i = L, R$, compete in a winner-take-all election by simultaneously and independently announcing (and committing to) a policy platform $x_i \in X = [0, 1]$. The electorate is made up of a continuum of voters. Each voter has a utility (loss) function $u_\theta(x) = -|x - \theta|$, where $\theta \in X$ denotes his preferred policy or *ideal point* on X. Due to the nature of voters' preferences (single-peaked and symmetric around θ), for every pair $(x_L, x_R) \in X^2$ each voter votes sincerely for the platform closer to its ideal point, voting for the alternatives with equal probabilities when indifferent.[7] Candidate i wins the election if his platform x_i gets more than half of the votes, with ties broken by a fair coin toss.

Apart from the uncertainty due to the possibility of a tie, candidates also have uncertainty about voters' preferences. We assume that the median voter's ideal point, denoted by θ_m, is uniformly distributed over $[1/2 - \beta, 1/2 + \beta]$, with $\beta > 0$. This may be because voters' preferences are fixed, but candidates perceive the fraction of types supporting their respective platforms with some noise, as happens for example in Roemer (2001, p. 45); or, because voters' preferences actually change after candidates have announced their platforms, as is the case in Bernhardt et al. (2009). Regardless of the interpretation given to the electoral uncertainty, it transpires from our assumptions that the probability that candidate L attaches to winning the election is given by $p(x_L, x_R) = \text{Prob}(\theta_m \in [0, \frac{x_L+x_R}{2}])$ if $x_L \leqslant x_R$, and by $p(x_L, x_R) = \text{Prob}(\theta_m \in [\frac{x_L+x_R}{2}, 1])$ if $x_L > x_R$. Candidate R's probability of winning is $1 - p(x_L, x_R)$.

As was said in the Introduction, candidates possess mixed or hybrid motives for running for office. Formally, the payoffs for candidate L and candidate R associated to any pair of policy platforms $(x_L, x_R) \in X^2$ are given by, respectively,

$$\Pi_L(x_L, x_R) = p(x_L, x_R) \cdot \bigl(u_{\theta_L}(x_L) + \chi_L\bigr) + \bigl[1 - p(x_L, x_R)\bigr] \cdot u_{\theta_L}(x_R), \tag{1}$$

and

$$\Pi_R(x_L, x_R) = \bigl[1 - p(x_L, x_R)\bigr] \cdot \bigl(u_{\theta_R}(x_R) + \chi_R\bigr) + p(x_L, x_R) \cdot u_{\theta_R}(x_L), \tag{2}$$

where θ_i stands for candidate i's ideological (preferred) position on X, and $\chi_i > 0$ denotes candidate i's payoff for being in power (office rents).[8] We assume that candidates' ideological positions are distributed on either side of the (expected) median voter's ideal policy, i.e., $\theta_L < 1/2 < \theta_R$; and we identify the half-open interval $[0, 1/2)$ (resp., $(1/2, 1]$) with the left-wing (resp., right-wing) candidate's ideological side. In addition, to rule out uninteresting equilibria with large electoral uncertainty and no trade-off between power and ideology, the essence of this investigation, we assume that $\beta < \bar{\beta} \equiv \min\{1/2 - \theta_L + \chi_L/2, \theta_R - 1/2 + \chi_R/2\}$.[9]

Let Δ be the space of probability measures on the Borel subsets of X. A mixed strategy for i is a probability measure $\mu_i \in \Delta$, with support $\text{supp}(\mu_i) \equiv \{x \in X\colon\ \forall \epsilon > 0, \mu_i((x - \epsilon, x + \epsilon) \cap X) > 0\}$. We extend each Π_i to Δ^2 by $U_i(\mu_L, \mu_R) = \int_{X^2} \Pi_i(x_L, x_R)\, d(\mu_L(x_L) \times \mu_R(x_R))$. Note that U_i is well defined because the set of discontinuities of Π_i, namely $\{(x_L, x_R) \in X^2\colon\ x_L = x_R \neq 1/2\}$, has measure zero.

[5] See Palfrey (2006) for a recent overview of these papers.

[6] See also Amaldoss and Jain (2002), Palacio-Huerta (2003), Palacio-Huerta and Volij (2008), and Levitt et al. (2010), among others.

[7] Since there are only two candidates and each of them enacts its proposed policy once elected, voting for the preferred candidate is a weakly dominant strategy for every voter.

[8] Note that Hotelling (1929)–Downs' (1957) office motivation hypothesis, according to which candidates maximize the probability of winning the election, is obtained by letting χ_i be arbitrarily large for all i. Likewise, Wittman's (1983) ideological candidates follow by setting the rents χ_i equal to zero.

[9] If that were not the case, then in an equilibrium with differentiated policies at least one candidate would maximize its payoff at its preferred location θ_i, independently of the position chosen by the other.

Let $\mathcal{G} = (X, \Pi_i)_{i=L,R}$ denote a *mixed motivation election game*, and let $\overline{\mathcal{G}} = (\Delta, U_i)_{i=L,R}$ be the mixed extension of $\mathcal{G}$. A Nash equilibrium of $\overline{\mathcal{G}}$ is a pair of probability measures $(\mu_L^*, \mu_R^*) \in \Delta^2$ such that for all $(x_L, x_R) \in X^2$, $U_L(\mu_L^*, \mu_R^*) \geqslant U_L(x_L, \mu_R^*)$ and $U_R(\mu_L^*, \mu_R^*) \geqslant U_R(\mu_L^*, x_R)$. We say that a Nash equilibrium $(\mu_L^*, \mu_R^*) \in \Delta^2$ is a mixed strategy equilibrium (MSE) of $\mathcal{G}$ if at least one candidate randomizes over two or more policies. Otherwise, if for all $i = L, R$, $\text{supp}(\mu_i^*) = \{x_i^*\}$ for some $x_i^* \in X$, then the profile (x_L^*, x_R^*) represents a pure strategy equilibrium (PSE) of $\mathcal{G}$.[10]

4. Equilibrium analysis

We begin the equilibrium analysis noting that $\mathcal{G}$ possesses neither a PSE where the left-wing candidate chooses a platform further to the right than the right-wing candidate's proposal, nor a PSE where one of the candidates wins the election for sure.

Lemma 1. *If the strategy profile $(x_L^*, x_R^*) \in X^2$ is a pure strategy equilibrium for the election game $\mathcal{G} = (X, \Pi_i)_{i=L,R}$, then $\theta_L < x_L^* \leqslant x_R^* < \theta_R$ and $p(x_L^*, x_R^*) \in (0, 1)$.*

The previous lemma, whose proof (as well as all other proofs) is given in Appendix A, is used to characterize each candidate's platform in a PSE with policy differentiation, and to provide a necessary condition for such an equilibrium to exist.

Lemma 2. *The election game $\mathcal{G} = (X, \Pi_i)_{i=L,R}$ has a pure strategy equilibrium with $x_L^* < x_R^*$ only if $\chi_L + \chi_R < 4\beta$, $x_L^* = 1/2 - \beta + \chi_L/2$, and $x_R^* = 1/2 + \beta - \chi_R/2$.*

The platforms characterized in Lemma 2 are a function of the electoral uncertainty β and the office rents χ_i, with the signs as expected. All the rest equal, as the candidates become less certain about how moderate the median voter is (higher β), they become more polarized in their platform choice. By contrast, a reduction of the uncertainty (resp., an increase of office rents) moves both platforms towards the center of the political space. Note, however, that these platforms are independent of the candidates' ideologies. Moreover, they are independent of each other too, in the sense that a change in candidate i's equilibrium policy x_i^* (due, for example, to a change in χ_i) does not affect x_j^*. These are mainly consequences of the linearity of the loss function.[11]

The platforms of Lemma 2 are obtained from the first-order conditions; that is, they are the stationary points of the conditional payoff functions. Unfortunately, Lemma 2 does not guarantee that these functions are quasi-concave. Therefore, a sensible question to ask is what additional conditions ensure the policy profile to be a Nash equilibrium. Propositions 2 and 3 are meant to shed some light into this inquiry. But first, we offer necessary and sufficient conditions for policy convergence (i.e., equilibrium with identical platforms), which is the classical result of electoral competition.

Proposition 1 *(Convergence). The election game $\mathcal{G} = (X, \Pi_i)_{i=L,R}$ has a pure strategy equilibrium with $x_L^* = x_R^* \equiv x^*$ if and only if $x^* = 1/2$ and $\chi_i \geqslant 2\beta$ for all $i = L, R$.*

One way of interpreting the condition specified in the statement of Proposition 1 is as follows. In this paper the winner enjoys an extra payoff for being elected equal to χ_i. From the candidates' viewpoint, however, hitting the median ideal point with a particular policy platform and actually winning the election has a chance of $(2\beta)^{-1}$ (the inverse of the length of the support of θ_m). Therefore, the term $\chi_i/2\beta$ can be viewed as the expected benefit for moving the platform one additional unit to the center (expected median). The cost of doing that is given by the additional unit of disutility created by the displacement towards the center and away from the candidate's ideology. Thus, when χ_i is large enough for all i (resp., β is small enough), in the sense that $\chi_i/2\beta \geqslant 1$, the benefits of any such deviation outweigh the costs and, consequently, candidates converge to the median voter's preferred policy.[12]

An immediate implication of Proposition 1 and Lemma 2 is the following corollary.

Corollary 1 *(Uniqueness). If the election game $\mathcal{G} = (X, \Pi_i)_{i=L,R}$ possesses a pure strategy equilibrium, then the equilibrium is unique.*

The uniqueness result expressed in Corollary 1 is more general than the related results found in Saporiti (2008) and Bernhardt et al. (2009), because the latter only refer to the homogeneous motivation case ($\chi_L = \chi_R$), whereas the former

[10] When $\mu \in \Delta$ assigns probability 1 to a single policy $x \in X$, we simply write x instead of μ.

[11] With a nonlinear loss function, equilibrium platforms would be interdependent and sensitive (directly or indirectly) to the ideology of each candidate.

[12] Two interesting instances where this occurs are: (i) when both candidates are *purely opportunistic*, which provides the standard median voter result of Hotelling (1929) and Downs (1957) (under certainty) and Calvert (1985) (under uncertainty); and (ii) when both candidates are *purely ideological* and they have perfect information about the median voter's location, as considered for example in Roemer (1994). As a matter of fact, in the latter case the result holds independently of candidates' motivations.

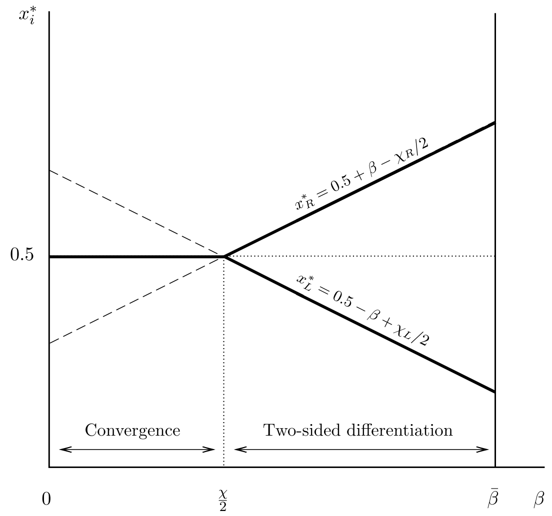


Fig. 1. Symmetric case: $\chi_L = \chi_R \equiv \chi$.

also applies to cases where χ_L is not necessarily equal to χ_R. It is worth reminding, however, that the three models are different and, therefore, that the results are not directly comparable.

The next proposition provides a necessary and sufficient condition for another well known equilibrium configuration (suggested first by Wittman, 1983, and proved later by Roemer, 1997), where each candidates chooses a policy on its own ideological side.

Proposition 2 *(Two-sided differentiation). The election game $\mathcal{G} = (X, \Pi_i)_{i=L,R}$ has a pure strategy equilibrium with $x_L^* < 1/2 < x_R^*$ if and only if $\chi_i < 2\beta$ for all $i = L, R$.*

Thus, the first conclusion that can be drawn by combining Propositions 1 and 2 is that, when candidates possess identical motivations, these two results offer a full description of the equilibrium outcomes. To illustrate this, Fig. 1 displays the equilibrium platforms as a function of the electoral uncertainty β, and for a particular level of office rents $\chi \equiv \chi_L = \chi_R$. As Proposition 1 points out, both policies are located at the estimated median voter's ideal point for any level of uncertainty lower than or equal to $\chi/2$. Above that threshold, Lemma 2 and Proposition 2 indicate that the equilibrium platforms lie down on each candidate's ideological ground, in accordance with the expressions $x_L^* = 1/2 - \beta + \chi_L/2$ and $x_R^* = 1/2 + \beta - \chi_R/2$. That gives rise to a region of two-sided policy differentiation as is shown in the graph. The symmetric location of the policies around the median also implies that, in the identical motivation case, the probability of winning is the same for the two candidates.

Interestingly, when candidates hold asymmetric interests, Propositions 1 and 2 do not cover the whole spectrum of possibilities. The main contribution of this paper is precisely to analyze what happens in that case. As we will show, besides the equilibria outlined above, there are other kind of equilibria that we will refer to as equilibria with *one-sided policy differentiation*. These equilibria are such that candidates locate on a different platform, but these platforms are on the same side of the median ideal point. When the right-wing candidate turns out to be the relatively more policy-concerned candidate, the conditions for one-sided differentiation are basically that the level of uncertainty be (i) sufficiently low to ensure that L's stationary point is above $1/2$; and (ii) high enough to discourage players to undercut their stationary points, ensuring in particular that $\limsup_{x_R \to x_L^*} \Pi_R(x_L^*, x_R) \leqslant \Pi_R(x_L^*, x_R^*)$. The interpretation of the conditions when the left-wing candidate is relatively more ideological is similar.

Proposition 3 *(One-sided differentiation). The election game $\mathcal{G} = (X, \Pi_i)_{i=L,R}$ has a pure strategy equilibrium with $1/2 < x_L^* < x_R^*$ (resp., $x_L^* < x_R^* < 1/2$) if and only if $(\chi_L - \chi_R)/2 + (\chi_R \cdot \chi_L)^{1/2} \leqslant 2\beta < \chi_L$ (resp., $(\chi_R - \chi_L)/2 + (\chi_R \cdot \chi_L)^{1/2} \leqslant 2\beta < \chi_R$).*

Postponing for the moment the interpretation of this result, we proceed by noting that apart from one-sided PSE, the asymmetric motivation case also admits equilibria in mixed strategies. To analyze the properties of these equilibria, the following notation is going to be helpful. First, denote the critical values of β stated in Proposition 3 by $\beta_1^C \equiv \frac{\chi_L - \chi_R}{4} + \frac{\sqrt{\chi_L \chi_R}}{2}$ and $\beta_2^C \equiv \frac{\chi_R - \chi_L}{4} + \frac{\sqrt{\chi_L \chi_R}}{2}$. Second, consider the region of the strategy space where $p(x_L, x_R) \in (0, 1)$. Within that region, for any $x_L' < 1/2 + \beta - \chi_R/2 = x_R^*$,

$$\Pi_R(x_L', x_R^*) = \frac{1}{4\beta}\left(\frac{1}{2} + \beta - x_L' + \frac{\chi_R}{2}\right)^2 + (x_L' - \theta_R),$$

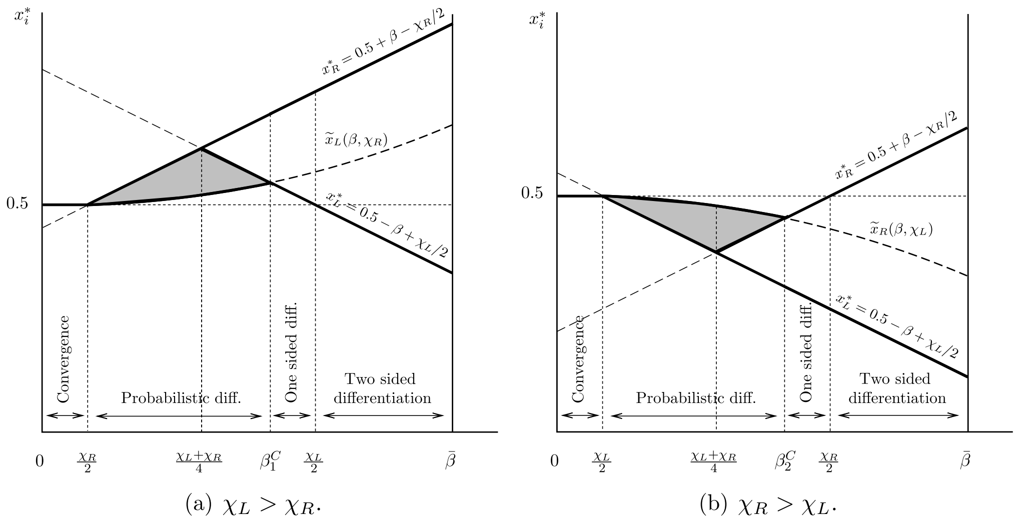


Fig. 2. Asymmetric case.

and

$$\limsup_{x_R \to^- x_L'} \Pi_R(x_L', x_R) = \left(\frac{1}{2} - \frac{1-2x_L'}{4\beta}\right)\chi_R + (x_L' - \theta_R).$$

Denote by $\widetilde{x}_L(\beta, \chi_R)$ the solution to $\Pi_R(x_L', x_R^*) - \limsup_{x_R \to^- x_L'} \Pi_R(x_L', x_R) = 0$.[13] The support of the mixed strategy equilibrium when the right-wing candidate is the relatively more ideological politician (see Fig. 2(a)) is characterized in the next proposition.[14]

Proposition 4 *(Probabilistic differentiation). If $\chi_R/2 < \beta < \beta_1^C$, the election game $\mathcal{G} = (X, \Pi_i)_{i=L,R}$ has a mixed strategy equilibrium $(\mu_L^*, \mu_R^*) \in \Delta^2$ with the property that,*

(a) *If $\beta \leqslant \frac{\chi_L+\chi_R}{4}$, then $\operatorname{supp}(\mu_i^*) = [\underline{x}, \bar{x}]$ for all $i = L, R$, with $\underline{x} = \widetilde{x}_L(\beta, \chi_R)$ and $\bar{x} = \frac{1}{2} + \beta - \frac{\chi_R}{2} = x_R^*$; and*
(b) *If $\beta > \frac{\chi_L+\chi_R}{4}$, then $\operatorname{supp}(\mu_L^*) = [\underline{x}, \bar{x}]$ and $\operatorname{supp}(\mu_R^*) = [\underline{x}, \bar{x}] \cup \{x_R^*\}$, with $\underline{x} = \widetilde{x}_L(\beta, \chi_R)$ and $\bar{x} = \frac{1}{2} - \beta + \frac{\chi_L}{2} = x_L^*$.*

Going back to the interpretation of the last two propositions, notice that in the asymmetric motivation case the more ideological candidate (henceforth "she") enjoys a "policy advantage," in the sense that the equilibrium policy ends up closer to what she prefers. That is because, given the uncertainty, she is more willing to take the risk of being close to her preferences. The opportunistic candidate (henceforth "he") is willing to follow her in order to increase his chances of winning the election, to which she reacts by randomizing on her side. However, when the uncertainty about the median voter is really high, the ideological candidate gets too close to her ideology, and the opportunistic guy is not willing to follow her that far in the policy space. This is what allows for differentiation in pure strategies on one side of the median. As a final observation, notice that so long as PSE policies differ, this case also predicts that the ideological candidate possesses a lower probability of winning the election; or, to put it differently, that the opportunist candidate enjoys an "electoral advantage."

To illustrate the results when candidates exhibit *asymmetric* motivations, we plot in Fig. 2 the equilibrium platforms as a function of the electoral uncertainty. As the graphs show, besides a range of low and high levels of uncertainty, when candidates possess heterogeneous interests it is also possible to distinguish a range of moderate or intermediate levels that provides distinct equilibrium predictions. The three levels of electoral uncertainty are determined by the following ranges of values of β:

1. *low uncertainty*: $0 \leqslant \beta \leqslant \min\{\frac{\chi_L}{2}, \frac{\chi_R}{2}\}$;
2. *moderate uncertainty*: $\min\{\frac{\chi_L}{2}, \frac{\chi_R}{2}\} < \beta < \max\{\frac{\chi_L}{2}, \frac{\chi_R}{2}\}$; and
3. *high uncertainty*: $\max\{\frac{\chi_L}{2}, \frac{\chi_R}{2}\} \leqslant \beta \leqslant \bar{\beta}$.

[13] To be precise, the solution turns out to be $\widetilde{x}_L(\beta, \chi_R) = 1/2 + \beta + 3/2\chi_R \pm \sqrt{2}\sqrt{2\beta\chi_R + \chi_R^2}$.

[14] An analogous characterization can be given for the case where the left-wing candidate is the relatively more ideological candidate (see Proposition 5 at the end of Appendix A).

Table 1
Experimental treatments.

Treatment	Uncertainty	Ideologies		Rents		NE Policies		NE Payoffs	
	β	θ_L	θ_R	χ_L	χ_R	x_L^*	x_R^*	L	R
1	2.5	10	90	10	10	50	50	550.0	550.0
2	15	10	90	10	10	40	60	550.0	550.0
3	15	10	90	40	40	50	50	700.0	700.0
4	15	34	66	10	10	40	60	790.0	790.0
5	35	10	90	10	10	20	80	550.0	550.0
6	15	10	90	90	10	MSE	MSE	1066.08	578.84
7	35	10	90	90	10	60	80	1064.29	664.29

As in the symmetric case, for low levels of uncertainty candidates converge to the estimated median voter's ideal point. However, as the length of the interval over which the median is distributed increases, there exists a range of intermediate levels of electoral uncertainty (namely, the values in Fig. 2(a) between $\chi_R/2$ and β_1^C, and the values in Fig. 2(b) between $\chi_L/2$ and β_2^C) for which the mixed motivation election game fails to possess an equilibrium in pure strategies. Within that region, labeled in the graphs *probabilistic differentiation*, the game admits an equilibrium in mixed strategies. Further, Proposition 4 (resp. Proposition 5 in Appendix A) states that the support of both candidates is on the same side of the median ideal point, as is illustrated by the grey area of Fig. 2(a) (resp. 2(b)).

As the electoral uncertainty continues increasing, it eventually surpasses either the critical threshold β_1^C if $\chi_L > \chi_R$, or the threshold β_2^C if $\chi_R > \chi_L$, and the existence of a pure strategy equilibrium is reestablished. For values of the uncertainty parameter above these thresholds and within the range of intermediate levels, Proposition 3 shows that a PSE not only exists, but also that the corresponding equilibrium policies are placed on the same ideological ground, giving rise to a region of *one-sided policy differentiation*. Afterwards, for high electoral uncertainty, the conditions of Proposition 2 hold, and each candidate chooses a policy on its own ideological side, although these policies do not locate symmetrically around the center.[15]

To conclude, we compute the payoffs associated with the different equilibria, showing how they vary with the relevant parameters. First, for policies converging to the median ideal point, the payoffs are $\Pi_L(x_L^*, x_R^*) = \frac{\chi_L}{2} + \theta_L - \frac{1}{2}$ and $\Pi_R(x_L^*, x_R^*) = \frac{\chi_R}{2} - \theta_R + \frac{1}{2}$, which are obviously increasing in candidates' own interest in power, and constant with respect to the electoral uncertainty.[16] In addition, note that the left-wing (resp. right-wing) candidate's payoff is increasing (resp. decreasing) in the candidate's ideology, since being closer to the expected median reduces the utility loss of moving away from the ideal point. Interestingly, the same equilibrium payoffs are obtained in the symmetric motivation case, regardless of whether the equilibrium is at the expected median or with two-sided policy differentiation.

Second, for differentiation in pure strategies with asymmetric interests, the left-wing candidate's equilibrium payoff is $\Pi_L(x_L^*, x_R^*) = \frac{\chi_L}{2} + \theta_L - \frac{1}{2} + \frac{(\chi_L - \chi_R)^2}{16\beta}$; and the right-wing candidate's is $\Pi_R(x_L^*, x_R^*) = \frac{\chi_R}{2} - \theta_R + \frac{1}{2} + \frac{(\chi_L - \chi_R)^2}{16\beta}$. These two depend on the ideologies as before; and they are decreasing in the electoral uncertainty, since higher uncertainty moves the policy location of the opportunistic candidate away from the relatively more policy-concerned one, and it also reduces the probability of the former of winning the election. Regarding the office rents, both equilibrium payoffs are increasing in their own interest in power[17]; and the cross effect is positive for the ideological candidate, but negative for the opportunistic one.[18] Finally, for differentiation in mixed strategies, we are unable to offer a general characterization of the payoffs due to the fact that we do not possess a closed form solution for the equilibrium distributions.

5. Experimental design

In this section, we present a laboratory experiment designed to assess the theoretical predictions of the mixed motivation election game studied in Section 4. The experiment consisted of seven treatments, which were determined by varying the uncertainty parameter β, the ideologies θ_i and the office rents χ_i. For the convenience of the experimental subjects we considered only integer locations, numbered from 0 to 100, which required multiplying the relevant parameter values for β, θ, and χ by 100. The values employed in each treatment, together with the corresponding equilibrium policies and

[15] As a matter of comparison, note that when $\chi_L = \chi_R$, all of the critical values of β indicated in Figs. 2(a) and 2(b) coincide, i.e., $\beta_1^C = \beta_2^C = \chi_R/2 = \chi_L/2$. That explains why Fig. 1 exhibits neither a region with a mixed strategy equilibrium, nor one with one-sided policy differentiation.

[16] Bear in mind that to get convenient values for the experiment, the payoffs of Table 1 include a positive constant of 90 in the utility function and a multiplication of the payoffs by 10.

[17] For $i, j = L, R$, $i \neq j$, $\frac{\partial \Pi_i(x_i^*, x_j^*)}{\partial \chi_i} = \frac{4\beta + \chi_i - \chi_j}{8\beta}$, which is positive because in Proposition 3 $\beta > \frac{\chi_i + \chi_j}{4}$.

[18] For $i, j = L, R$, $i \neq j$, $\frac{\partial \Pi_i(x_i^*, x_j^*)}{\partial \chi_j} = \frac{\chi_j - \chi_i}{8\beta}$, which is positive if $\chi_j > \chi_i$ and non-positive otherwise.

Table 2
MSE policies for treatment 6.

Support	Left-wing candidate		Right-wing candidate	
	density	c.d.f.	density	c.d.f.
52	0.5529	0.5529	0.0919	0.0919
53	0.1048	0.6577	0.0117	0.1036
54	0.2295	0.8872	0.0409	0.1445
55	0.0000	0.8872	0.0000	0.1445
56	0.0887	0.9759	0.0225	0.1670
57	0.0000	0.9759	0.0000	0.1670
58	0.0229	0.9988	0.0117	0.1788
59	0.0012	1.0000	0.0000	0.1788
60	0.0000	1.0000	0.8212	1.0000

payoffs, are displayed in Table 1. For Treatment 6, with a MSE, we report the expected equilibrium payoffs, and the reader is referred to Table 2 for details of the MSE policies.[19]

Subjects were told in the instructions a brief story of a town holding a two candidate, majority rule election to select the location of a new post office on the high street. The subjects' task was to propose simultaneously and independently an integer number between 0 and 100 to locate the post office. They knew that voters were distributed uniformly across the 101 locations, and they were told that although each voter would vote for the proposal closer to its own location, for each profile of proposed locations the percentage of votes received by each candidate was not known with certainty due to the existence of some uncertainty about voters' preferences.

Subjects were also informed about the preferred location on the high street for each of the two candidates. In order to get convenient payoff values in the lab, we applied a linear transformation adding, first, a positive constant of 90 to the loss function; and then multiplying payoffs by 10. Subjects were told that they would receive a location payoff corresponding to 900 minus 10 times the distance between their ideal location (θ) for the post office and the location actually realized. In addition, subjects were told that winning the election would provide to the winning candidate an extra payoff of $\chi \cdot 10$.[20]

The locations were chosen by typing in a number on the decision screen. A screenshot of the interface is provided in Fig. 3. Before making their actual proposals, subjects were provided with the opportunity to use an *expected payoff calculator* (top half of the screen) in which they could enter several hypothetical locations for themselves and for their opponent and calculate the associated own payoff. This calculator offered subjects a convenient device for looking at the 101×101 payoff matrix, but it makes no recommendation as how to play the game. There was no time limit for subjects' decisions.

After all participants made their actual choices, in each round subjects found a feedback screen with their chosen location, the location chosen by the other candidate, and the resulting own payoff, denominated in points. Subjects were recommended to transcribe the results of each round from the feedback window on a provided logsheet.

In each treatment there were 2 or 3 sessions, each comprising 60 rounds (elections). At the beginning of each session, subjects were randomly and anonymously matched into pairs. Within each pair, one subject was assigned the role of candidate A, whereas the other played the role of candidate B. Subjects were informed that they would not know who of the other people in the room they were paired with, and that matched pairs would remain fixed for the entire session. They were also aware that their initial roles would be swapped after round 30. This swapping allowed us to study some aspects of the learning by the subjects, particularly the transfer of insights from one role to the other. It also removed possible concerns about payoff asymmetries present in some of the treatments.

The experiment was carried out in the Spring of 2010 in the Centre for Experimental Economics of the University of York. Subjects were recruited from a university-wide pool of undergraduate and postgraduate students using Greiner's (2004) Online Recruitment System for Economic Experiments (ORSEE). The experiment was programmed and conducted with the software Z-Tree (Fischbacher, 2007).

Upon arrival, subjects were assigned to a computer terminal and they were given a set of written instructions.[21] After reading the instructions, they were allowed to ask questions by raising their hands and speaking with the experimenter in private. To ensure that subjects understood the decision situation and the mechanics of payoff calculations, all participants answered several computerized test questions. The experiment did not proceed until every subject had answered these questions correctly. Subjects were not allowed to communicate directly with one another, and they only interacted indirectly via the decisions they entered in the computer terminals.

[19] The computations were done with the software GAMBIT (McKelvey et al., 2010). Obviously, there are differences between the (discrete) numerical results of Table 2 and the (continuous) theoretical predictions of Proposition 4. However, these differences vanish as the grid becomes finer.

[20] Note that we framed the experiment as a game of electoral competition, informing the subjects about the two different motivations for the candidates, while avoiding potentially confounding political left–right connotations. The reason for this framing is that we wanted to test precisely whether in an election game as analyzed in the theory the subjects can learn to play the equilibrium strategies.

[21] A copy of the instructions is available in a supplementary online appendix.

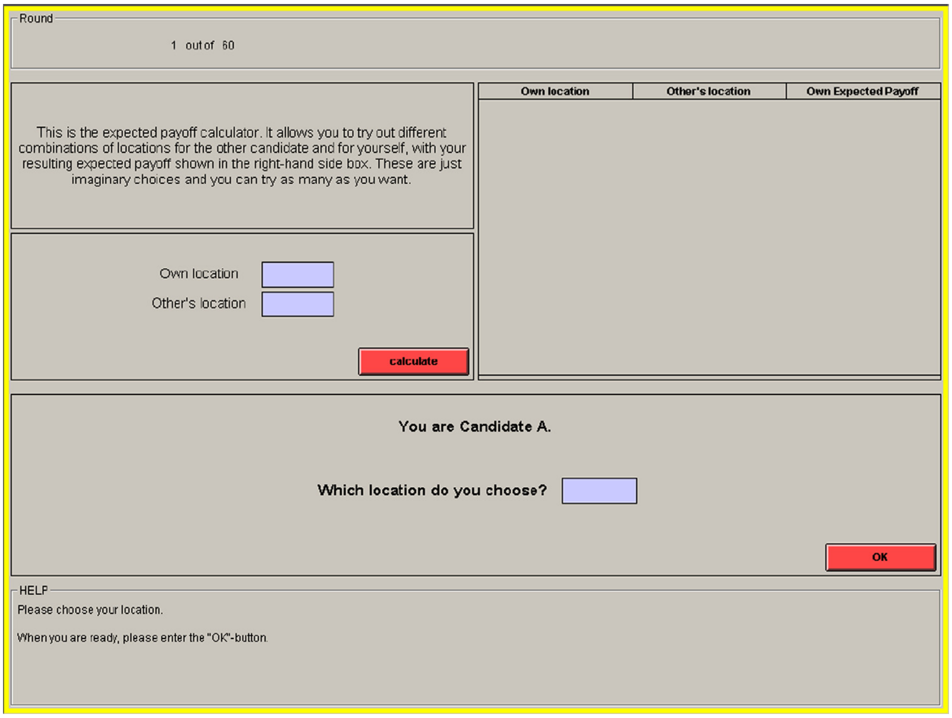


Fig. 3. Decision interface.

Table 3
Overview of the experiment.

Treatment	Sessions	Subjects	Pairs	Exchange rate (GBP per 1000 points)	Average payment (GBP)
1	3	26	13	0.60	19.80
2	2	20	10	0.60	19.80
3	2	20	10	0.50	21.00
4	2	20	10	0.45	21.30
5	2	20	10	0.60	19.81
6	2	20	10	0.40	24.56
7	3	30	15	0.40	24.85

Subjects were informed that the points accumulated throughout the 60 rounds would determine, together with a given exchange rate, their monetary payoffs. A typical session lasted approximately 2 hours. The average payment in each treatment, the exchange rate, and the number of sessions, participants, and pairs are all summarized in Table 3.

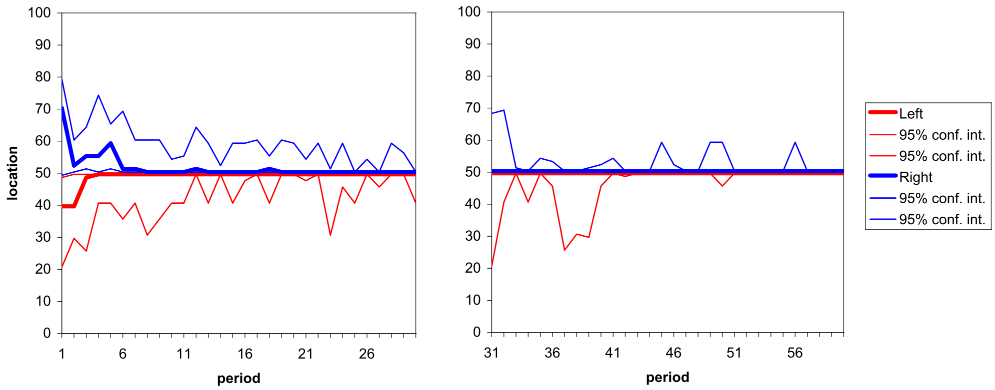
6. Experimental evidence

6.1. Equilibrium convergence

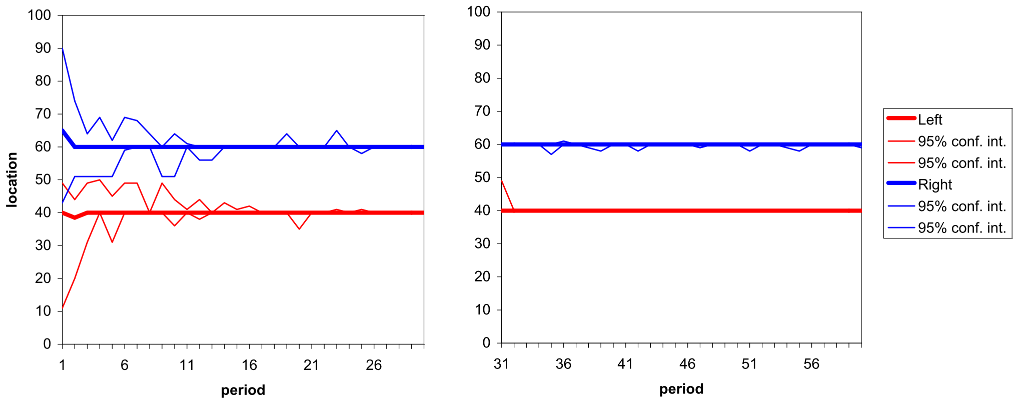
First, we look at the location choices of the Left and the Right players in the various treatments, and we compare them with the Nash equilibrium values. The supplementary online appendix displays disaggregated data on these variables for single periods, for subintervals of the 60 periods, and for matching pairs.

Fig. 4 shows for each treatment for which a PSE exists the per period median location of the Left and the Right players, as well as the 95% confidence intervals. These confidence intervals are determined as follows. Depending on the treatment, for each period there are between ten and fifteen independent observations (pairs). Using these observations as the unit of analysis, for every possible value m between 0 and 100, we test the null hypothesis (two-sided binomial test) that m is the median, i.e., that the probability to observe a location choice below m equals the probability to observe one above m. The alternative hypothesis is that the median has either a lower or a higher value than m, i.e., that these probabilities are not equal. For any given value m, the null hypothesis is rejected if there are too few or too many observations on one side of m.

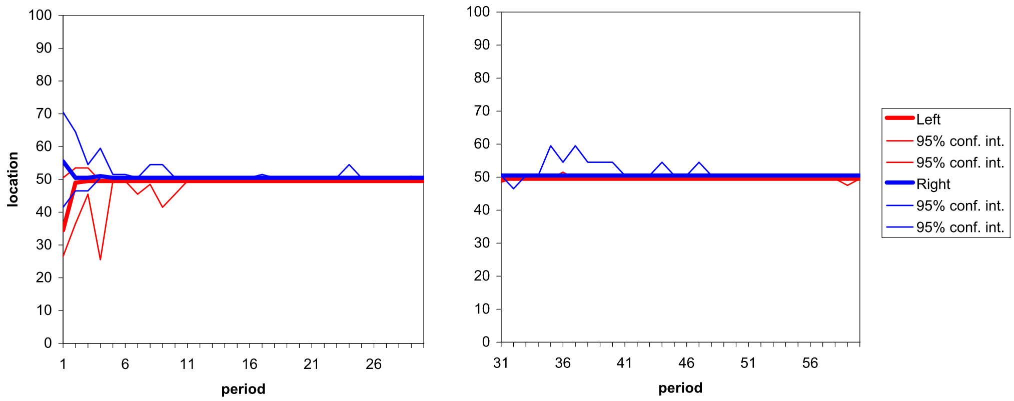
Two main conclusions emerge from the graphs. On the one hand, in Treatments 1 to 5 (see Figs. 4(a)–4(e)) not only the median locations converge to the equilibrium values, but also the 95% confidence intervals shrink over time. On the other hand, in Treatment 7 (see Fig. 4(f)) with one-sided differentiation, although the median locations of the Left and the Right



(a) Treatment 1: $x_L^* = x_R^* = 50$.



(b) Treatment 2: $x_L^* = 40$ & $x_R^* = 60$.

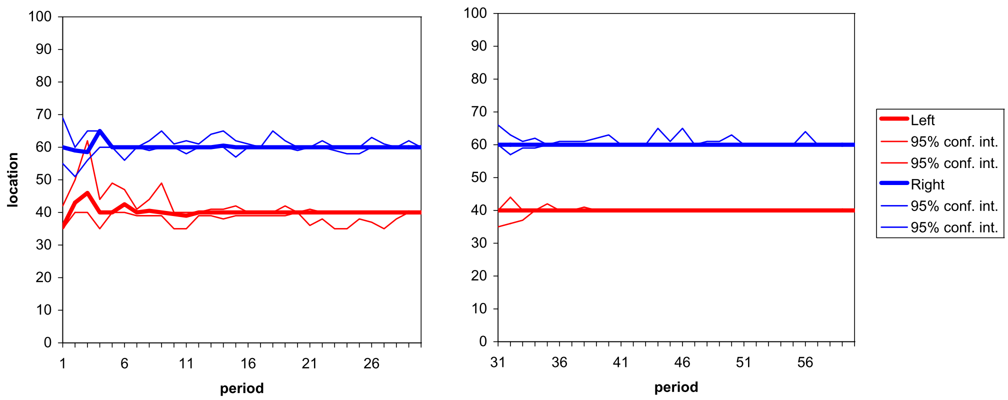


(c) Treatment 3: $x_L^* = x_R^* = 50$.

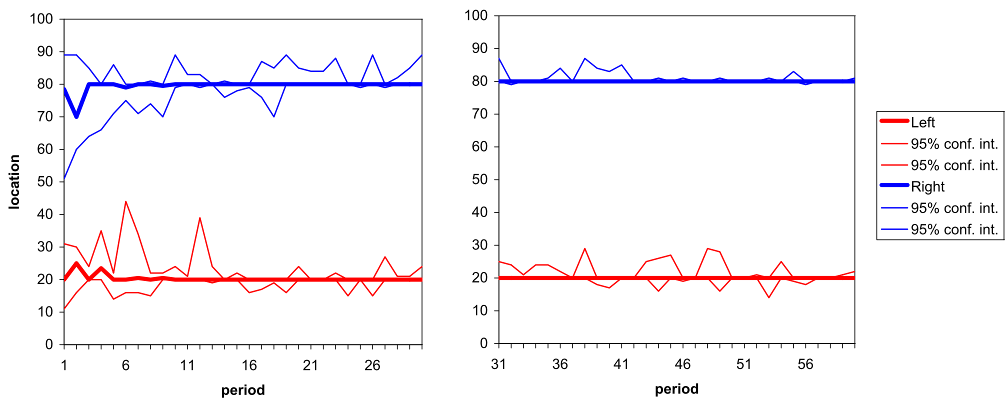
Fig. 4. Median locations and 95% confidence intervals.

players converge to the equilibrium, the 95% confidence intervals of both players tend to be skewed towards the center of the policy space. This suggests that although most of the players behaved in the lab as the theory predicts, some Left as well as some Right players deviated and they tended to stay towards the left of the theoretical predictions and closer to the center even after 60 periods of play.

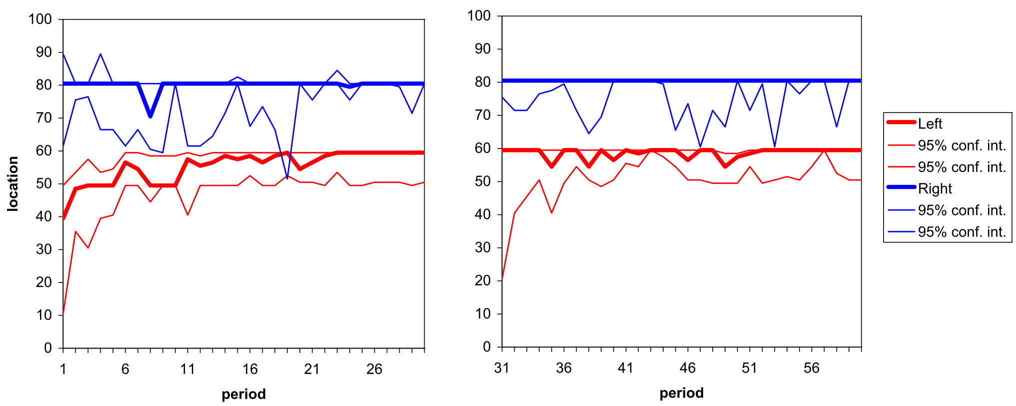
As to Treatment 6, notice that this case is different because the unique Nash equilibrium of the game is in mixed strategies. Therefore, besides the median locations of the Left and the Right players, in Figs. 5(a) and 5(b) we also display for each period the minimum and the maximum values of their locations, and we compare these values with the theoretical lower and upper bounds of the MSE support.



(d) Treatment 4: $x_L^* = 40$ & $x_R^* = 60$.



(e) Treatment 5: $x_L^* = 20$ & $x_R^* = 80$.

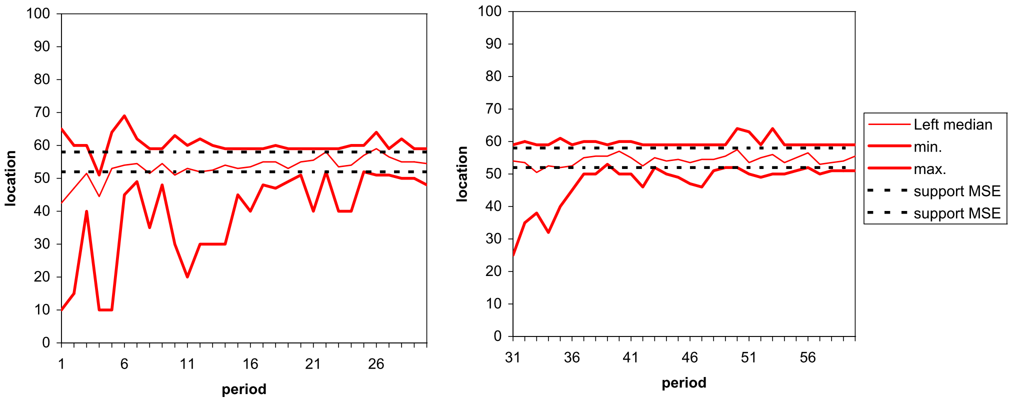


(f) Treatment 7: $x_L^* = 60$ & $x_R^* = 80$.

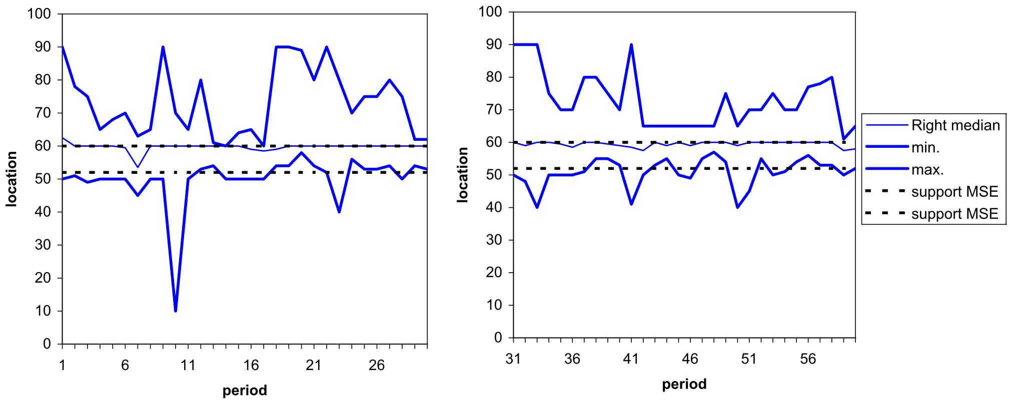
Fig. 4. (*continued*).

We find that the median of the Left (resp. Right) players converges to 55 (resp. 60), which is close to (resp. coincides with) the median location of the MSE (52 and 60, for Left and Right players respectively). Moreover, the pictures show that the minimum and the maximum locations chosen in the lab approximate the bounds of the MSE support, which ranges from 52 to 59 for the Left player, and from 52 to 60 for the Right player.

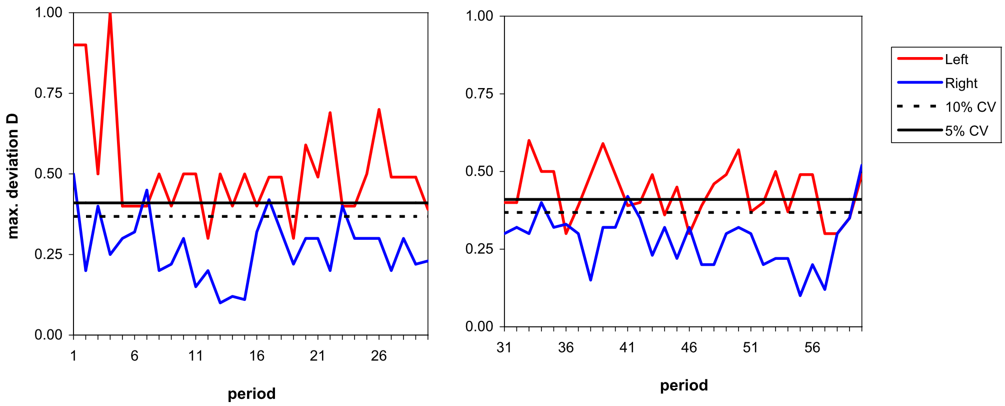
Since the median and the support measure just some aspects of the distributions, to further assess the differences between the empirical and the theoretical distributions, we apply the Kolmogorov–Smirnov test, considering for each period ten independent observations for the Left players and ten observations for the Right players. The test statistic, denoted by D, represents the maximum deviation between the empirical and the theoretical cumulative distributions. The null hypothesis



(a) Median, minimum and maximum locations of the Left players.



(b) Median, minimum and maximum locations of the Right players.

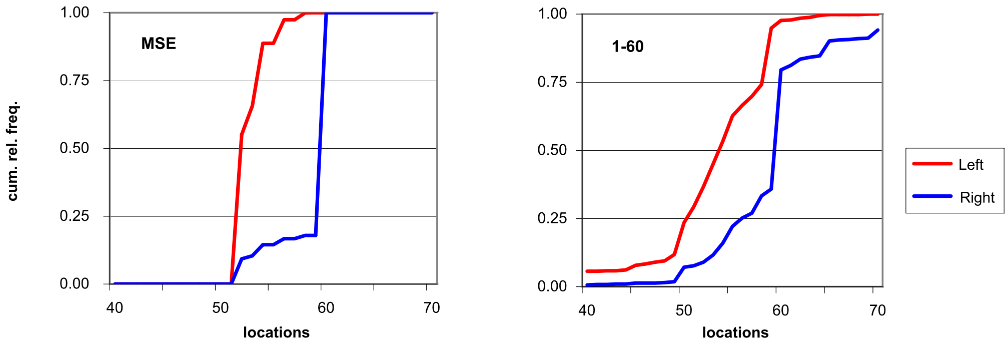


(c) Maximum deviation from the cumulative equilibrium distributions in each period.

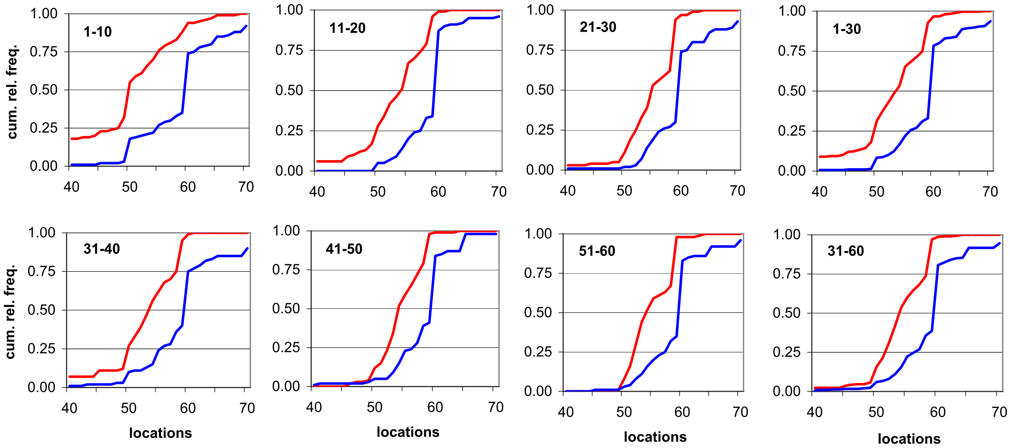
Fig. 5. Treatment 6.

is that these distributions are identical. The alternative hypothesis is that they are not the same. The critical values to reject the null hypothesis at 5% and 10% significance levels are, respectively, 0.410 and 0.368 (see Siegel, 1988), with values of D above the critical values leading to the rejection of the null hypothesis.

For each of the 60 periods separately, Fig. 5(c) shows the test statistic D for the Left and the Right players as well as the critical values (CV). As we see, we cannot reject the null hypothesis in most of the periods for the Right players. Specifically, using the 5% critical value, the MSE distribution cannot be rejected in 27 of the first 30 periods, and 28 of the last 30 periods. For the Left players, however, the picture is somewhat different. Still at 5% significance, the MSE distribution cannot be rejected in 11 periods in the first half of the experiment, and 15 periods in the second half.



(d) Cumulative relative frequencies.



(e) Cumulative relative frequencies in different subintervals.

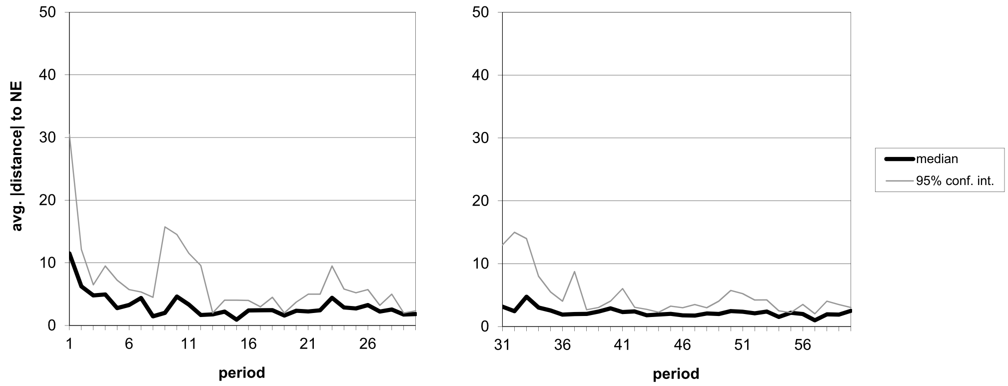
	max. dev. D		critical values	
	Left	Right	5%	10%
1-10	0.59	0.26	0.14	0.12
11-20	0.38	0.16	0.14	0.12
21-30	0.50	0.26	0.14	0.12
1-30	0.37	0.22	0.08	0.07
31-40	0.33	0.25	0.14	0.12
41-50	0.37	0.23	0.14	0.12
51-60	0.37	0.17	0.14	0.12
31-60	0.35	0.21	0.08	0.07
1-60	0.35	0.21	0.06	0.05

(f) Kolmogorov-Smirnov test.

Fig. 5. (*continued*).

In Figs. 5(d) and 5(e) we continue the analysis of Treatment 6, presenting the empirical cumulative distributions for the 60 period interval as a whole and for a number of different subintervals. In conformity with the theory, the graphs show that the cumulative distribution of the Left players first-order stochastically dominates the distribution of the Right players. But when the Kolmogorov–Smirnov test is applied to these subintervals of the 60 periods (see Fig. 5(f)), we see that the null hypothesis of the empirical distributions being indistinguishable from the MSE distributions must be rejected in every single case. This means that the empirical distributions of the Left and the Right players are indeed statistically different from the theoretical ones.

The question, then, is how substantial these differences are. To answer that question, in every period we take the empirical distribution of the ten Left (Right) players, and we compute for each of these players how many locations they would need to move to reach the MSE distribution (allowing for fractions of players). To do this, we only stretch, squash and shift the empirical distribution, thus preserving the order of the location choices. That is, if player i had chosen a location smaller (greater) than player j, then after all moves have been made to reach the MSE distribution, player i still has a location smaller (greater) than or equal to player j.



(g) Median average absolute distance from the MSE distribution (with 95% confidence interval).

Fig. 5. (*continued*).

Table 4
Players' median locations (significance levels for rejection of H_0).

	treat1	treat2	treat3	treat4	treat5	treat6	treat7
H_0	$L=R$	$L=R$	$L=R$	$L=R$	$L=R$	$L=R$	$L=R$
H_1	$L<>R$	$L<R$	$L<>R$	$L<R$	$L<R$	$L<R$	$L<R$
Periods							
1–10	1% (<)	0.1%	1% (<)	0.1%	0.1%	0.1%	0.00%
11–20	1% (<)	0.1%	no diff.	0.1%	0.1%	0.1%	0.00%
21–30	10% (<)	0.1%	1% (<)	0.1%	0.1%	0.1%	0.00%
1–30	1% (<)	0.1%	1% (<)	0.1%	0.1%	0.1%	0.00%
31–40	10% (<)	0.1%	no diff.	0.1%	0.1%	0.1%	0.00%
41–50	no diff.	0.1%	no diff.	0.1%	0.1%	0.1%	0.00%
51–60	10% (<)	0.1%	no diff.	0.1%	0.1%	0.1%	0.00%
31–60	10% (<)	0.1%	no diff.	0.1%	0.1%	0.1%	0.00%
1–60	1% (<)	0.1%	5% (<)	0.1%	0.1%	0.1%	0.00%

Once the number of locations each player would need to move to reach the equilibrium distribution has been found, in any given period we take the average number of moves of the Left and the Right player in each matching pair as the distance between the empirical and the theoretical distributions. This provides for each period ten independent observations for this distance. Fig. 5(g) shows that the median distance as well as the 95% confidence interval diminish over time, and that in the last subinterval, i.e., in periods 51–60, on average the median distance to be moved is only 2.0 locations. This means that although the empirical distributions of the Left and the Right players are statistically different from the theoretical ones, these differences are relatively small.

Up to this point we focused our analysis of the experimental data on a comparison with the Nash equilibrium predictions.[22] Interestingly, in some treatments the Left and Right players are predicted to converge to the same location, whereas in others the equilibrium predictions for Left and Right players are different. Therefore we now turn to a comparison of the positions of the Left players with the positions of the Right players. For each treatment and each matching pair, we compute the average position of the Left and of the Right players in different intervals. Thus, depending on the treatment, for each interval we have ten to fifteen independent observations, each of them being a matched pair. We use the Wilcoxon signed-ranks test to assess whether we can reject the null hypothesis that the position of the Left players is equal to that of the Right players. The results (one- or two-tailed tests as indicated by H_1) are shown in Table 4.

As we see, in each treatment where the Left players would be expected to be on the left of the Right players (i.e., in Treatments 2, 4, 5, 6, and 7) this was indeed what happened. Note that in Treatment 6, it can happen according to the MSE predictions that a Left player chooses a location to the right of the Right player, because the supports of the equilibrium distributions overlap. Nevertheless, for each of the intervals considered the expected mean location for the Left player is to the left of that of the Right player.

In Treatments 1 and 3 the Left and the Right players were supposed to converge to the same location. Nevertheless, the table shows that the position of the Left players was often significantly to the left of that of the Right players in these two treatments. Note that although statistically significant, these deviations were not widespread, as was shown above in Fig. 4 by the convergence of the medians to the Nash equilibrium. In as far as there were deviations from the PSE in these

[22] We also considered Quantal Response Equilibria (QRE). For each treatment we estimated the QRE choice intensity parameter by minimizing the error of the QRE strategy profiles with the empirical distribution observed in periods 41–60. Using this free parameter we obtain errors for the QRE that are only marginally below those for the Nash equilibrium predictions, and this slightly better fit is achieved by using widely different choice intensity parameter values across treatments.

Table 5
Players' average payoffs (significance levels for rejection of H_0).

	treat1	treat2	treat3	treat4	treat5	treat6	treat7
Left	552.2	550.6	704.2	788.2	545.9	1052.65	1027.92
Right	547.8	549.4	695.8	791.3	553.7	584.64	629.54
H_0	$L = R$	$L = R$	$L = R$	$L = R$	$L = R$	$L = R$	$L = R$
H_1	$L <> R$	$L <> R$	$L <> R$	$L <> R$	$L <> R$	$L > R$	$L > R$
Periods 1–60	no diff.	no diff.	no diff.	no diff.	no diff.	1% (>)	1% (>)

treatments, they tended to be towards the left for Left players and towards the right for Right players. This may be explained by a bias induced by the subjects' ideology, or by the out-of-equilibrium incentives.[23]

Finally, regarding the equilibrium payoffs, Table 1 shows that in the symmetric Treatments 1–5, both players get equal payoffs. On the contrary, the asymmetry in the parameter values of Treatments 6 and 7 creates an asymmetry in the equilibrium payoffs as well, with the more opportunistic Left player getting higher payoffs in equilibrium than her opponent with lower office rents. One question, then, is whether this payoff asymmetry materializes in the experiment as well.

Table 5 shows for each treatment the average payoffs of the Left and of the Right players over the 60 periods.[24] We use the Wilcoxon signed-ranks test to assess whether we can reject the null hypothesis that the payoff of the Left players is equal to that of the Right players. The results (one- or two-tailed tests as indicated by H_1) show that the null hypothesis of equal payoffs cannot be rejected in Treatments 1 to 5. Moreover, as predicted by the theory, in Treatments 6 and 7 the payoffs of the Left players are significantly greater than those of the Right players at 1% significance level.

6.2. Learning

Having studied the convergence of the subjects' choices to the equilibrium, we now examine in which periods this convergence takes place. For each treatment, we distinguish the 30 periods before the swapping of the roles and the 30 periods after the swap. We also split these intervals into smaller subintervals of ten periods. For every matching pair, we compute for each subinterval the average absolute distance from the Nash equilibrium, and we test whether these distances are different in two specified intervals.

To do this, we use the one-tailed Wilcoxon signed-ranks test, distinguishing 1% and 5% significance levels. This is a non-parametric statistical test to assess whether there is a difference in the median of two related samples. The only assumption made about the underlying distribution is that these differences are independent observations from a symmetric distribution. The null hypothesis is that the median difference between the pairs of observations is zero. The alternative hypothesis is that the median of the interval that comes later is lower than that of the earlier interval, reflecting the learning and adaptive behavior of the experimental subjects.

The results are reported in Table 6. In each box, we compare the average absolute distance in the intervals indicated on the left-hand side to those indicated at the top of the box. Thus, if we consider for instance Treatment 1 (first box), we see that the average absolute distance from the PSE is smaller in periods 11–20 (first column at the top) than in periods 1–10 (first row on left-hand side) at the 1% significance level. For Treatment 6 we present two boxes: the first box (treat6a) shows the distance from the MSE support, whereas the second (treat6b) shows the distance from the entire distribution.

First, we ask whether there has been a significant amount of learning over the entire experiment. As the tables show, learning did happen since in every treatment the average absolute distance from the Nash equilibrium is statistically significantly smaller in the last ten periods, i.e., in periods 51–60, than in the first ten periods.

Second, we ask in which periods the average absolute distance actually decreases. Looking at the main diagonal of the tables, it turns out that except in Treatment 7, where it seems that learning happened between periods 11 and 20, in the rest of the treatments learning took place mainly in the first ten periods (elections), which was also the most active interval in terms of subjects' use of the expected payoff calculator.[25]

Third, we ask whether players after swapping their roles between periods 30 and 31 succeed in transferring some of their findings from before the swapping to after the swapping. The answer is largely affirmative as the distance from the Nash equilibrium is smaller in periods 31–40 than in periods 1–10 for all treatments except Treatment 1.

Finally, we test whether the swapping as such led to an increase in the distance from the NE right after the swapping. As we see in Table 7, in some treatments there is an increase in the distance from the equilibrium if the intervals considered are 1 or 5 periods before the swap, but not considering a ten period interval.

[23] If the opponent chooses the PSE location, then deviating from the PSE towards a subject's own ideology leads to a less steep decline in payoffs than a deviation in the opposite direction.

[24] The average payoffs for each matching pair can be found in the supplementary online appendix.

[25] These findings are confirmed by OLS regressions, where the position of the Left and the Right players and the average absolute distance from the equilibrium are regressed against the inverse of time $1/t$ as the only independent variable (see the supplementary online appendix). The analysis shows that almost all coefficient are significant; and, in particular, the slope coefficients for the distance from the equilibrium are significant and with the expected sign for all treatments.

Table 6
Decrease in the average absolute distance from the Nash equilibrium.

From periods	To periods					
	11–20	21–30	31–40	41–50	51–60	31–60
treat1						
1–10	yes (1%)	yes (1%)	no	yes (1%)	yes (1%)	
11–20		no	no	no	yes (5%)	
21–30			no	no	yes (1%)	
31–40				yes (1%)	yes (1%)	
41–50					no	
1–30						yes (1%)
treat2						
1–10	yes (1%)	yes (1%)	yes (1%)	yes (1%)	yes (1%)	
11–20		no	no	no	no	
21–30			no	no	no	
31–40				no	no	
41–50					no	
1–30						yes (1%)
treat3						
1–10	yes (1%)	yes (1%)	yes (5%)	yes (1%)	yes (1%)	
11–20		no	no	no	no	
21–30			no	no	no	
31–40				yes (1%)	yes (5%)	
41–50					no	
1–30						no
treat4						
1–10	yes (1%)	yes (1%)	yes (1%)	yes (1%)	yes (1%)	
11–20		no	no	no	yes (1%)	
21–30			no	no	no	
31–40				no	yes (5%)	
41–50					no	
1–30						yes (1%)
treat5						
1–10	yes (1%)	yes (1%)	yes (1%)	yes (5%)	yes (5%)	
11–20		no	no	no	no	
21–30			no	no	no	
31–40				no	no	
41–50					no	
1–30						yes (5%)
treat6a						
1–10	yes (1%)	yes (1%)	yes (5%)	yes (1%)	yes (1%)	
11–20		no	no	no	no	
21–30			no	no	no	
31–40				yes (5%)	yes (5%)	
41–50					no	
1–30						yes (5%)
treat6b						
1–10	yes (1%)	yes (1%)	yes (5%)	yes (1%)	yes (5%)	
11–20		no	no	no	yes (1%)	
21–30			no	yes (10%)	yes (5%)	
31–40				yes (5%)	yes (1%)	
41–50					no	
1–30						yes (5%)
treat7						
1–10	no	yes (1%)	yes (5%)	yes (1%)	yes (1%)	
11–20		yes (1%)	no	yes (5%)	yes (1%)	
21–30			no	no	no	
31–40				no	yes (5%)	
41–50					no	
1–30						yes (1%)

6.3. *Comparisons between treatments*

In Sections 6.1 and 6.2, we compared for each treatment separately the experimental data with the Nash equilibrium values. We now perform a number of 'comparative statics' tests across these treatments. For expositional convenience, all the pair-wise comparisons are illustrated in Fig. 6, where a double arrow relating any two treatments is used to indicate a direct statistical comparison between them.

Table 7
Increase in the average absolute distance from the Nash equilibrium.

From period	To period			From period	To period		
	31–40	31–35	31		31–40	31–35	31
treat1				treat5			
21–30	no			21–30	no		
26–30		no		26–30		no	
30			yes (5%)	30			no
treat2				treat6a			
21–30	no			21–30	no		
26–30		no		26–30		no	
30			no	30			yes (5%)
treat3				treat6b			
21–30	no			21–30	no		
26–30		yes (5%)		26–30		no	
30			yes (5%)	30			yes (5%)
treat4				treat7			
21–30	no			21–30	no		
26–30		no		26–30		no	
30			no	30			no

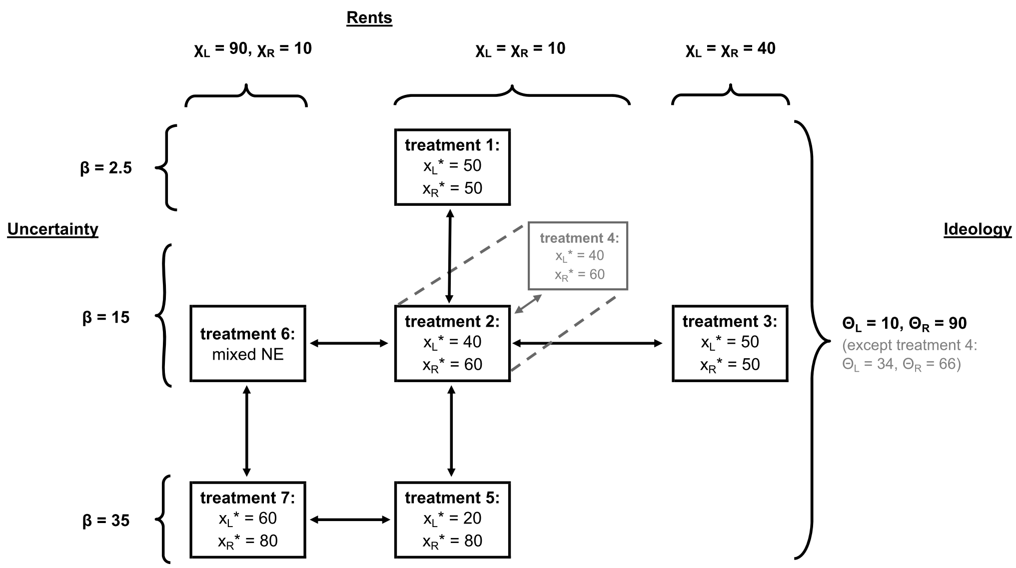


Fig. 6. Overview of the comparisons between treatments.

The comparative statics tests carried out here focus mainly on three variables: the position of the Left players, the position of the Right players, and the average absolute distance from the Nash equilibrium. In each treatment, we compute the average value of these variables for different subintervals and for the whole session. We have, depending on the treatment, between ten and fifteen independent observations, and we use the robust rank-order test to compare the samples between two treatments, distinguishing 1%, 5% and 10% significance levels.[26] The results found are reported in Table 8. Table 8(a) concerns the positions of the Left players, Table 8(b) the location of the Right players, and Table 8(c) shows the average absolute distance from the Nash equilibrium.

First, to assess the impact on policy divergence of an increase in the electoral uncertainty, Treatment 1 is compared with Treatments 2 and 5, respectively, and Treatment 2 is compared with Treatment 5. In each of these treatments, the ideologies and the office rents remain constant, whereas the electoral uncertainty gradually increases, leading to increasing policy divergence in theory. The results are shown in the second, third and fourth columns of Table 8(a)–(c). In conformity with the theory, in all cases and in every interval the null hypothesis that there is no difference between the positions of

[26] This test statistic has the advantage that it compares the median of two unrelated samples without making any assumptions about the higher moments of the distribution of the two samples. The critical values are taken from Feltovich (2003).

Table 8
Differences between treatments.

	Treatments							
	1 v. 2	2 v. 5	1 v. 5	2 v. 4	2 v. 3	2 v. 6	5 v. 7	6 v. 7
H_0	$L(1) = L(2)$	$L(2) = L(5)$	$L(1) = L(5)$	$L(2) = L(4)$	$L(2) = L(3)$	$L(2) = L(6)$	$L(5) = L(7)$	$L(6) = L(7)$
H_1	$L(1) > L(2)$	$L(2) > L(5)$	$L(1) > L(5)$	$L(2) <> L(4)$	$L(2) < L(3)$	$L(2) < L(6)$	$L(5) < L(7)$	$L(6) < L(7)$
Periods								
1–10	5%	1%	1%	no diff.	1%	1%	1%	no diff.
11–20	1%	1%	1%	no diff.	1%	1%	1%	no diff.
21–30	1%	1%	1%	no diff.	1%	1%	1%	no diff.
1–30	1%	1%	1%	no diff.	1%	1%	1%	no diff.
31–40	5%	1%	1%	no diff.	1%	1%	1%	no diff.
41–50	1%	1%	1%	no diff.	1%	1%	1%	10%
51–60	1%	1%	1%	no diff.	1%	1%	1%	no diff.
31–60	1%	1%	1%	no diff.	1%	1%	1%	no diff.
1–60	1%	1%	1%	no diff.	1%	1%	1%	no diff.

(a) Left players' positions across treatments.

	Treatments							
	1 v. 2	2 v. 5	1 v. 5	2 v. 4	2 v. 3	2 v. 6	5 v. 7	6 v. 7
H_0	$R(1) = R(2)$	$R(2) = R(5)$	$R(1) = R(5)$	$R(2) = R(4)$	$R(2) = R(3)$	$R(2) = RL(6)$	$R(5) = R(7)$	$R(6) = R(7)$
H_1	$R(1) < R(2)$	$R(2) < R(5)$	$R(1) < R(5)$	$R(2) <> R(4)$	$R(2) > R(3)$	$R(2) <> R(6)$	$R(5) <> R(7)$	$R(6) < R(7)$
Periods								
1–10	5%	1%	1%	no diff.	1%	no diff.	no diff.	1%
11–20	1%	1%	1%	no diff.	1%	no diff.	no diff.	1%
21–30	1%	1%	1%	no diff.	1%	no diff.	no diff.	1%
1–30	1%	1%	1%	no diff.	1%	no diff.	no diff.	1%
31–40	1%	1%	1%	no diff.	1%	no diff.	1% (>)	1%
41–50	1%	1%	1%	no diff.	1%	no diff.	1% (>)	1%
51–60	1%	1%	1%	no diff.	1%	no diff.	no diff.	1%
31–60	1%	1%	1%	no diff.	1%	no diff.	1% (>)	1%
1–60	1%	1%	1%	no diff.	1%	no diff.	5% (>)	1%

(b) Right players' positions across treatments.

	Treatments							
	1 v. 2	2 v. 5	1 v. 5	2 v. 4	2 v. 3	2 v. 6	5 v. 7	6 v. 7
H_0	$d(1) = d(2)$	$d(2) = d(5)$	$d(1) = d(5)$	$d(2) = d(4)$	$d(2) = d(3)$	$d(2) = d(6)$	$d(5) = d(7)$	$d(6) = d(7)$
H_1	$d(1) <> d(2)$	$d(2) <> d(5)$	$d(1) <> d(5)$	$d(2) <> d(4)$	$d(2) <> d(3)$	$d(2) <> d(6)$	$d(5) <> d(7)$	$d(6) <> d(7)$
Periods								
1–10	no diff.	no diff.	no diff.	no diff.	no diff.	no diff.	no diff.	1% (<)
11–20	no diff.	no diff.	no diff.	no diff.	no diff.	no diff.	5% (<)	10% (<)
21–30	10% (>)	no diff.	no diff.	no diff.	no diff.	5% (<)	10% (<)	no diff.
1–30	no diff.	no diff.	no diff.	no diff.	no diff.	no diff.	5% (<)	5% (<)
31–40	10% (>)	no diff.	no diff.	no diff.	no diff.	2% (<)	5% (<)	no diff.
41–50	no diff.	no diff.	no diff.	no diff.	no diff.	2% (<)	10% (<)	no diff.
51–60	no diff.	no diff.	no diff.	no diff.	no diff.	1% (<)	no diff.	no diff.
31–60	10% (>)	no diff.	no diff.	no diff.	no diff.	2% (<)	10% (<)	no diff.
1–60	no diff.	10% (<)	no diff.	no diff.	no diff.	5% (<)	5% (<)	10% (<)

(c) Average absolute distance from the Nash equilibrium across treatments.

the Left (resp. Right) players across the treatments is rejected at 1% or 5% significance levels, with the alternative hypothesis being in the direction predicted by the theory.

As to the average absolute distance from the Nash equilibrium, the tests indicate no significant differences in most of the intervals. However, looking at the whole session, Treatment 5 appears to show less convergence than Treatment 2, albeit only at 10% significance level. We conjecture that the reason could be that the equilibrium associated with the parameter values of Treatment 5 (i.e., $x_L^* = 20$ and $x_R^* = 80$) is somewhat more extreme than the one corresponding to Treatment 2 (i.e., $x_L^* = 40$ and $x_R^* = 60$), and that some of the subjects may have been concerned about choosing such extreme policies.[27]

[27] In our analysis of the experimental data we assume risk-neutrality of the experimental subjects.

Table 9
Players' average payoffs (significance levels for rejection of H_0).

	treat2 v. treat6		treat5 v. treat7	
	Left	Right	Left	Right
H_0	$L(2) = L(6)$	$R(2) = R(6)$	$L(5) = L(7)$	$R(5) = R(7)$
H_1	$L(2) < L(6)$	$R(2) < R(6)$	$L(5) < L(7)$	$R(5) < R(7)$
Periods 1–60	1%	1%	1%	1%

Second, by varying the ideologies, the comparison of Treatments 2 and 4 offers the chance to see whether the two-sided differentiation effect present in Treatment 2 is independent of the degree of ideological polarization $\theta_R - \theta_L$.[28] In conformity with the theory, in every interval the null hypothesis that there is no difference between the positions of the Left (resp. Right) players and between the average absolute distances cannot be rejected at 1% and 5% significance levels.

Third, the issue of whether policy convergence is reestablished as candidates become more office-motivated is investigated by comparing Treatments 2 and 3. The results show that in every interval the positions of the Left (resp. Right) players in Treatment 2 are statistically different at 1% significance level from the positions of the Left (resp. Right) players in Treatment 3, which is again consistent with the theory. Moreover, there are no statistically significant differences in these two treatments with respect to the average absolute distances from the Nash equilibrium.

Fourth, to assess the change in policy differentiation that results from raising the office rents of one of the candidates while keeping the other constant, Treatment 5 is contrasted with Treatment 7. The theory predicts no changes in the location of the Right candidate, and a move of the Left candidate from the left-hand side to the right-hand side of the median voter. The experimental results are mixed. On the one hand, in every interval the positions of the Left players in Treatment 7 are statistically different at 1% significance level from the positions of the Left players in Treatment 5. On the other hand, contrary to the theoretical prediction, we find significant differences in the Right players' positions in several intervals, including the last 30 periods (at %1) and the whole session (at 5%). The data show that the locations of these players in Treatment 5 tend to be more extreme. Consistent with our previous results, convergence to the NE is also worse in Treatment 7 than in Treatment 5. In the whole session as well as in several subintervals, there are significant differences (at 5 and 10%) in the average absolute distances from the NE, with the distance in Treatment 5 tending to be smaller.

Fifth, we compare Treatment 6, in which there is no PSE, with Treatments 2 and 7, to detect any significant variations in subjects' behavior in the absence of a PSE. For a start, comparing Treatment 6 with Treatment 2, we observe that the Left players in the latter, in which office rents are lower, choose locations to the left of those in Treatment 6. For Right players we do not see a difference between these two treatments, which seems related to the fact that the expected median in Treatment 6 is 59 whereas in Treatment 2 is 60. Next, comparing Treatment 6 with Treatment 7, in which uncertainty has increased, we see that there are no significant differences in the Left players' behavior (recall the expected median in the former is 53 and in the latter 60); whereas the Right players, as predicted, choose locations more to the right in Treatment 7. Finally, although with a small number of observations per period one cannot expect to hit the equilibrium distribution exactly in Treatment 6, we see that in the first twenty periods the distance from the equilibrium is nevertheless smaller in Treatment 6 than in Treatment 7.

Sixth, we study the effect of asymmetric office rents on the players' payoffs, comparing the payoffs of Treatments 2 and 6, and of Treatments 5 and 7. Table 1 shows that, as the office rents for the Left player increase in Treatment 6 (resp. 7), his payoffs become higher than in Treatment 2 (resp. 5). Moreover, the payoffs of the Right player become higher than in Treatment 2 (resp. 5) as well. As was explained in Section 4, this happens because as the more opportunistic player moves into the ideological side of his opponent, this makes it more likely that he will win the election, offering an electoral advantage that increases his payoffs. Simultaneously, his opponent is better off too because the policy implemented gets closer to her ideology. This is the ideological advantage of the relatively more policy-concerned candidate, which offers her higher equilibrium payoffs.

We use the robust rank-order test to compare the payoffs between these treatments over periods 1–60, and to test ultimately the electoral and the ideological advantage effects. As we can see from Table 9, introducing an asymmetry in the payoff parameters led in all cases to differences in players' payoffs that are consistent with the theoretical predictions at 1% significance level.

7. Final remarks

This paper builds on the spatial literature of electoral competition, studying theoretically and experimentally the set of Nash equilibria when candidates are interested in power and ideology, but not necessarily in the same way. It provides

[28] According to the theory, given the assumption of Euclidean preferences, the only effect of the ideologies on the equilibrium policies is through expanding or contracting the region of policy differentiation. Specifically, that region shrinks as the difference between the θs gets smaller (i.e., with less polarization).

a full characterization of the set of Nash equilibria, showing how the equilibrium configurations depend on the relative interests in power (resp., ideology) and the uncertainty about voters' preferences. In addition, it examines the empirical content of these theoretical predictions through a series of laboratory treatments. The experimental data show convergence to the equilibrium values at the aggregate and at the individual levels in all treatments, and comparative statics effects across treatments consistent with the theory. What is more, learning happens relatively quickly, especially if one takes into account that the experimental subjects had no experience of and received no further information about electoral games.

Despite these positive results, and despite the fact that the model considered here seems rich enough to pick up several interesting features of electoral competition that had been overlooked in the literature, there are a number of issues that may require more attention in future work. First, the assumption of risk-neutrality (with respect to the distance $|x - \theta_i|$), embedded into the assumption of Euclidean preferences of Section 3, entails a loss of generality in the analysis. This is because in spite of being ideologically different, risk averse candidates tend to move closer to each other and toward to the center.[29] We conjecture that the uniqueness of our equilibria and the different types of equilibrium configurations identified in Section 4 might be a property of elections that hold under a more general class of utility functions and electoral uncertainty. However, a full analysis of this conjecture and a complete equilibrium characterization under different conditions of preferences and uncertainty are beyond the scope of this paper.

Second, we noted in the experimental evidence that convergence to the Nash equilibrium is not equally precise across treatments, with the convergence being least precise in the asymmetric treatments, i.e., when the two candidates have different motives. As predicted, individual decisions were more noisy in the treatment with the MSE; and matching the exact probability distribution seems a more demanding test of convergence as well. But also in the asymmetric treatment with PSE, convergence was less precise than in the symmetric treatments. It will be interesting to investigate the causes of this difference in the degree of convergence across treatments, and to find out, for example, whether this observation that there is less convergence in the asymmetric treatments is due to the fact that the theoretical predictions implied one-sided policy differentiation, or just to the fact that these equilibria are not symmetric around the center. Further experiments may shine some light on this matter.

Third, an important element of our experimental design is the expected payoff calculator. The calculator provided information about the available payoffs. Such information is usually presented in the form of a payoff matrix in experimental settings. We had not made the entire 101×101 payoff matrix available for practical reasons. Instead, the calculator allowed the subjects to observe snapshots of the underlying payoff matrix. However, this did not create any bias, in the sense that it did not induce the subjects to examine any particular areas of the strategy space. Subjects had to enter explicitly the location choices for themselves and for their opponents, and the calculator only provided factual information about the corresponding payoffs, without suggesting any kind of recommendation. Having said that, it may be interesting to consider alternative designs in this type of electoral games, in particular designs in which information about the strategic environment is conveyed in a different way to the subjects.

Finally, as is conventional in the literature, our experimental design treats voters as artificial actors. It would be interesting, however, to organize an experiment in which the voters are experimental subjects as well. This has been done in some of the early papers about the median voter outcome, and it should be easier to implement nowadays thanks to the communication tools (such as smartphones, iPads, etc.) currently available. This may be interesting from a methodological viewpoint, as well as to assess related issues not modeled in the current work, such as private polling and voter turnout.

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Appendix A. Proofs

To simplify the notation, and given that the term $u_{\theta_i}(x_j)$, $i \neq j$, of candidate i's payoff function Π_i defined in (1) and (2) does not affect i's optimal choices, in the rest of this appendix we work with the linear transformations $\pi_i(x_i, x_j) \equiv \Pi_i(x_i, x_j) - u_{\theta_i}(x_j)$.

[29] Indeed, given the position of one candidate, the rival chooses a less differentiated platform when it is risk averse because it must compensate a higher utility loss due to the risk aversion with a rise in the probability of winning.

Proof of Lemma 1. Let (x_L^*, x_R^*) be a PSE for $\mathcal{G}$. To see that $p(x_L^*, x_R^*) \in (0,1)$, assume without loss of generality that $p(x_L^*, x_R^*) = 1$. Then, candidate R's equilibrium payoff is $\pi_R(x_L^*, x_R^*) = 0$; and it would be possible for R to increase its payoff by deviating to x_L^* (which would result in a payoff equal to $\chi_R/2 > 0$), a contradiction.

Next, suppose that $x_L^* < \theta_L$. If $x_R^* \geqslant \theta_L$, it would be possible for L to increase its payoff by choosing θ_L, because $\pi_L(x_L^*, x_R^*) = p(x_L^*, x_R^*) \cdot [x_L^* + x_R^* - 2\theta_L + \chi_L] < p(\theta_L, x_R^*) \cdot [x_R^* - \theta_L + \chi_L] = \pi_L(\theta_L, x_R^*)$.[30] Alternatively, if $x_R^* < \theta_L$, then: (i) L would profitably deviate to x_R^* if $x_L^* < x_R^*$, because $\pi_L(x_L^*, x_R^*) = p(x_L^*, x_R^*) \cdot [x_L^* - x_R^* + \chi_L] < \chi_L/2$; (ii) R would find it beneficial to move to x_L^* if $x_R^* < x_L^*$, because $\pi_R(x_L^*, x_R^*) = [1 - p(x_L^*, x_R^*)] \cdot [x_R^* - x_L^* + \chi_R] < \chi_R/2$; and (iii) L would do better by playing θ_L if $x_R^* = x_L^*$, because $\chi_L/2 < p(\theta_L, x_R^*) \cdot [\theta_L - x_R^* + \chi_L] = \pi_L(\theta_L, x_R^*)$. Therefore, $x_L^* \geqslant \theta_L$.

Assume, by way of contradiction, that $x_L^* = \theta_L$. Then: (i) if $x_R^* = \theta_L$, candidate R can benefit by moving its proposal to $x_R = \theta_L + \delta$, with $\delta > 0$ small, because $\pi_R(x_L^*, x_R) = [1 - p(x_L^*, x_R)] \cdot (\chi_R + \delta) > \chi_R/2 = \pi_R(x_L^*, x_R^*)$; (ii) if $x_R^* > \theta_L$, candidate L would be able to increase its payoff by selecting $x_L = \theta_L + \epsilon$, which would result, given the assumption on β and for $\epsilon > 0$ small enough, in a positive payoff change $[p(x_L, x_R^*) - p(\theta_L, x_R^*)] \cdot [x_R^* - \theta_L + \chi_L] - \epsilon \cdot p(x_L, x_R^*)$[31]; finally (iii) if $x_R^* < \theta_L$, R would find it profitable to deviate to θ_L because $\pi_R(x_L^*, x_R^*) = [1 - p(x_L^*, x_R^*)] \cdot (x_R^* - x_L^* + \chi_R) < \chi_R/2$. Hence, from (i)–(iii), we conclude that $x_L^* > \theta_L$. A similar argument establishes that $x_R^* < \theta_R$.

To complete the proof, it remains to be shown that $x_L^* \leqslant x_R^*$. Assume, by way of contradiction, that $x_L^* > x_R^*$. There are three cases to consider.

Case 1. If $x_R^* \in [0, \theta_L)$, candidate L can deviate to θ_L (recall that $x_L^* > \theta_L$), which results in a payoff change equal to $\pi_L(\theta_L, x_R^*) - \pi_L(x_L^*, x_R^*) = [p(\theta_L, x_R^*) - p(x_L^*, x_R^*)] \cdot [\theta_L - x_R^* + \chi_L] + p(x_L^*, x_R^*) \cdot (x_L^* - \theta_L) > 0$, contradicting that x_L^* is candidate L's best response to x_R^* (again $p(\theta_L, x_R^*) - p(x_L^*, x_R^*) > 0$ because of the monotonicity of $p(\cdot)$).

Case 2. If $x_R^* \in [\theta_L, 1/2)$, then L can deviate to $x_L = x_R^* + \epsilon$, $\epsilon > 0$, which results in a payoff change equal to $\pi_L(x_L, x_R^*) - \pi_L(x_L^*, x_R^*) = p(x_L, x_R^*) \cdot (\chi_L - \epsilon) - p(x_L^*, x_R^*) \cdot [\chi_L - (x_L^* - x_R^*)]$. By the properties of $p(\cdot)$ mentioned before, $p(x_L, x_R^*) \geqslant p(x_L^*, x_R^*)$. Thus, for ϵ small enough, $\pi_L(x_L, x_R^*) > \pi_L(x_L^*, x_R^*)$, implying that L's deviation is profitable and, consequently, that (x_L^*, x_R^*) is not a PSE; a contradiction.

Case 3. Finally, if $x_R^* \in [1/2, \theta_R)$, then $p(x_L^*, x_R^*) < 1/2$; and L can achieve a payoff greater than $\pi_L(x_L^*, x_R^*) = p(x_L^*, x_R^*) \cdot [\chi_L - (x_L^* - x_R^*)]$ by choosing x_R^* (which actually offers a payoff of $\chi_L/2$), contradicting the initial hypothesis that (x_L^*, x_R^*) is a PSE.

Therefore, from Cases 1–3, we conclude that $x_L^* \leqslant x_R^*$, as required. □

Proof of Lemma 2. Let the profile $(x_L^*, x_R^*) \in X^2$, with $x_L^* < x_R^*$, be a PSE for $\mathcal{G}$. By Lemma 1, $\theta_L < x_L^* < x_R^* < \theta_R$ and $p(x_L^*, x_R^*) \in (0,1)$. Since the probability function $p(\cdot)$ is continuous at (x_L^*, x_R^*), there must exist $\epsilon > 0$ sufficiently small such that, for all $(x_L, x_R) \in R_\epsilon(x_L^*) \times R_\epsilon(x_R^*)$, $\theta_L < x_L < x_R < \theta_R$ and $p(x_L, x_R) \in (0,1)$, where $R_\epsilon(x_i^*) \equiv (x_i^* - \epsilon, x_i^* + \epsilon)$, with $i = L, R$. Thus, for any profile $(x_L, x_R) \in R_\epsilon(x_L^*) \times R_\epsilon(x_R^*)$, the left-wing candidate's payoff function can be written as $\pi_L(x_L, x_R) = p(x_L, x_R) \cdot (x_R - x_L + \chi_L)$, where $p(x_L, x_R) = 1/2 + (x_L + x_R - 1)/4\beta$.

Fix $x_R^* \in R_\epsilon(x_R^*)$ and consider candidate L's best response to x_R^* over $R_\epsilon(x_L^*)$, which is obtained by solving the problem $\max_{x_L \in R_\epsilon(x_L^*)} \pi_L(x_L, x_R^*)$. The first-order condition for this problem provides a stationary point $1/2 - \beta + \chi_L/2$. Note that this point actually maximizes $\pi_L(\cdot, x_R^*)$ over $R_\epsilon(x_L^*)$ because by hypothesis, for all $x_L \in R_\epsilon(x_L^*)$, $\pi_L(x_L^*, x_R^*) \geqslant \pi_L(x_L, x_R^*)$; i.e., $\pi_L(\cdot, x_R^*)$ has an interior maximum on $R_\epsilon(x_L^*)$. Moreover, since $\pi_L(\cdot, x_R^*)$ is *strictly* concave on $R_\epsilon(x_L^*)$, with $\partial^2 \pi_L(x_L, x_R^*)/\partial x_L^2 = -1/2\beta < 0$, we have that $x_L^* = 1/2 - \beta + \chi_L/2$, as required. A similar argument shows that $x_R^* = 1/2 + \beta - \chi_R/2$.

Finally, the condition $x_L^* > \theta_L$ (resp., $x_R^* < \theta_R$) is obtained from the early assumption about β, (namely, $0 < \beta < \min\{1/2 - \theta_L + \chi_L/2,\ \theta_R - 1/2 + \chi_R/2\}$), whereas the condition $\chi_L + \chi_R < 4\beta$ follows from the initial hypothesis, according to which $x_L^* < x_R^*$. Routine calculations also show that $\chi_L + \chi_R < 4\beta$ implies that $(x_L^* + x_R^*)/2 \in (1/2 - \beta, 1/2 + \beta)$, so that $p(x_L^*, x_R^*) \in (0,1)$ as needed. □

Proof of Proposition 1. To show sufficiency, fix the strategy profile $(x_L^*, x_R^*) = (1/2, 1/2)$, where both candidates propose the median voter's ideal point and receive a payoff of $\pi_i(x_L^*, x_R^*) = \chi_i/2$. Consider first a deviation for the left-wing candidate to any platform $x'_L \in (\theta_L, 1/2)$. For convenience, let's write $x'_L = 1/2 - \delta$, with $\delta > 0$. Routine calculations show that $\pi_L(x'_L, x_R^*) \equiv \frac{\chi_L}{2} - \frac{\delta^2}{4\beta} + (\frac{1}{2} - \frac{\chi_L}{4\beta})\delta > \chi_L/2$ if and only if $\delta < 2\beta - \chi_L$. However, the last inequality requires $\delta < 0$ because by hypothesis $\chi_L \geqslant 2\beta$. Hence, $\pi_L(x'_L, x_R^*) \leqslant \pi_L(x_L^*, x_R^*)$. A similar argument proves that for any $x'_R \in (1/2, \theta_R)$, $\pi_R(x_L^*, x'_R) \leqslant \pi_R(x_L^*, x_R^*)$. The careful reader should also check at this point that any deviation above $1/2$ or below θ_L (resp., below $1/2$ or above θ_R) cannot raise candidate L's (resp., R's) conditional payoff any further, proving in that way that the profile $(x_L^*, x_R^*) = (1/2, 1/2)$ is a PSE for $\mathcal{G}$.

[30] Bear in mind that $p(\theta_L, x_R^*) \geqslant p(x_L^*, x_R^*)$, since for any two platforms $x_L < x_R$ (resp., $x_L > x_R$), $p(x_L, x_R)$ is non-decreasing (resp., non-increasing) in x_i, for all $i = L, R$. We use this property of $p(\cdot)$ several times in the rest of this proof.

[31] Note that $p(\theta_L, x_R^*) \in (0,1)$ because by hypothesis $x_L^* = \theta_L$. Hence, $p(x_L, x_R^*) - p(\theta_L, x_R^*) > 0$.

To show necessity, fix a PSE for $\mathcal{G}$ with the property that $x_L^* = x_R^* \equiv x^*$ for some $x^* \in X$. If $x^* > 1/2$, then candidate L can profitably deviate to $1/2$, because $p(1/2, x^*) \in (1/2, 1]$ and therefore $\pi_L(1/2, x^*) = p(1/2, x^*) \cdot [x^* - 1/2 + \chi_L] > 1/2 \cdot \chi_L = \pi_L(x^*, x^*)$. A similar reasoning shows that candidate R can profitably deviate to $1/2$ if $x^* < 1/2$. Therefore, $x^* = 1/2$.

Next, suppose that $\chi_L < 2\beta$, which in turn implies that $1/2 + \chi_L/2 - \beta < 1/2$. Since $p(\cdot)$ is continuous at $(1/2, 1/2)$ and strictly positive, there must exist $\delta > 0$ such that for all $x_L \in (1/2 - \delta, 1/2]$, $p(x_L, 1/2) > 0$ and $\pi_L(x_L, 1/2) = (\frac{1}{2} + \frac{x_L - 1/2}{4\beta}) \cdot (1/2 - x_L + \chi_L)$. Simple calculations show that $\pi_L(\cdot, 1/2)$ achieves a unique maximum over $(1/2 - \delta, 1/2]$ at $\hat{x}_L = 1/2 + \chi_L/2 - \beta$, implying in particular that $\pi_L(\hat{x}_L, 1/2) > \pi_L(1/2, 1/2)$, a contradiction. Hence, $\chi_L \geqslant 2\beta$. A similar argument proves that $\chi_R \geqslant 2\beta$. $\square$

Proof of Proposition 2. To prove necessity, suppose $\mathcal{G}$ has a PSE with the property that $x_L^* < 1/2 < x_R^*$. By Lemma 2, $x_L^* = \frac{1}{2} - \beta + \frac{\chi_L}{2}$ and $x_R^* = \frac{1}{2} + \beta - \frac{\chi_R}{2}$. Therefore, using the initial hypothesis, it follows that $\chi_i < 2\beta$ for all $i = L, R$.

To show sufficiency, fix the equilibrium candidate $(x_L^*, x_R^*) = (\frac{1}{2} - \beta + \frac{\chi_L}{2}, \frac{1}{2} + \beta - \frac{\chi_R}{2})$. By the initial hypothesis, i.e., $\chi_i < 2\beta$ for all $i = L, R$, it follows that $x_L^* < 1/2 < x_R^*$, $\chi_L + \chi_R < 4\beta$, and $p(x_L^*, x_R^*) \in (0, 1)$. By the assumption on β, $\theta_L < x_L^*$ and $x_R^* < \theta_R$. Applying the reasoning of the proof to Lemma 2, for some $\epsilon > 0$ such that $R_\epsilon(x_L^*) \equiv (x_L^* - \epsilon, x_L^* + \epsilon) \subset (\theta_L, x_R^*)$, we have that $x_L^* = \arg\max_{x_L \in R_\epsilon(x_L^*)} \pi_L(x_L, x_R^*)$, with $\pi_L(x_L^*, x_R^*) = \frac{\chi_L}{2} + (\beta - \frac{\chi_R}{2}) + \frac{(\chi_L - \chi_R)^2}{16\beta}$. Thus, $\pi_L(x_L^*, x_R^*) > \chi_L/2 = \pi_L(x_R^*, x_R^*)$.

Consider a deviation for the left-wing candidate to any platform $x_L' \in [0, 1]$ different from x_L^* and x_R^*. On one hand, if $p(x_L', x_R^*) = 0$, then $\pi_L(x_L', x_R^*) = 0 < \pi_L(x_L^*, x_R^*)$, implying that the alternative policy does not raise L's payoff. On the other hand, if $p(x_L', x_R^*) \in (0, 1]$, two cases are in order:

Case 1. Assume $x_L' \in (x_R^*, 1]$. Then: (i) if $p(x_L', x_R^*) = 1$, it must be the case that $1 - (x_L' + x_R^*)/2 \geqslant 1/2 + \beta$, which leads to the contradiction $(x_L' - 1/2) + (\beta - \chi_R/2) \leqslant -2\beta$, since the left-hand side of the previous inequality is strictly positive and the right-hand side is smaller than zero; alternatively (ii) if $p(x_L', x_R^*) \in (0, 1)$, then $\pi_L(x_L', x_R^*) = (\frac{1}{2} + \frac{1 - x_L' - x_R^*}{4\beta}) \cdot (x_R^* - x_L' + \chi_L)$. Recall that $1 - x_L' - x_R^* < 0$ and $x_R^* - x_L' < 0$, because $x_L' > x_R^* > 1/2$. Therefore, $\pi_L(x_L', x_R^*) < 1/2 \cdot \chi_L < \pi_L(x_L^*, x_R^*)$, implying once again that candidate L's deviation to x_L' is not beneficial.

Case 2. Suppose $x_L' \in [0, x_R^*)$. Then: (i) if $p(x_L', x_R^*) = 1$, it must be that $(x_L' + x_R^*)/2 \geqslant 1/2 + \beta$ and, consequently, that $x_L' \geqslant 1/2 + \beta + \chi_R/2 > x_R^*$, which supplies the desired contradiction (because by hypothesis $x_L' < x_R^*$); alternatively (ii) if $p(x_L', x_R^*) \in (0, 1)$, then: (ii.a) if $\theta_L \leqslant x_L' < x_R^*$, candidate L's deviation payoff is $\pi_L(x_L', x_R^*) = (\frac{1}{2} + \frac{x_L' + x_R^* - 1}{4\beta}) \cdot (x_R^* - x_L' + \chi_L)$; and, given that the function $f(x_L) = (\frac{1}{2} + \frac{x_L + x_R^* - 1}{4\beta}) \cdot (x_R^* - x_L + \chi_L)$ is strictly concave on $x_L \in [\theta_L, x_R^*)$ and has a maximum at $1/2 - \beta + \chi_L/2$, we conclude that $\pi_L(x_L', x_R^*) < \pi_L(x_L^*, x_R^*)$; finally (ii.b) if $0 \leqslant x_L' < \theta_L$, it is easy to show that $\pi_L(x_L', x_R^*) < \pi_L(\theta_L, x_R^*) < \pi_L(x_L^*, x_R^*)$, where the last inequality follows from the argument in (ii.a).

Summing up, Case 1 and Case 2 above, together with the fact that $\pi_L(x_L^*, x_R^*) > \pi_L(x_R^*, x_R^*)$, prove that $x_L^* = \arg\max_{x_L \in [0,1]} \pi_L(x_L, x_R^*)$. A similar reasoning also shows that $x_R^* = \arg\max_{x_R \in [0,1]} \pi_R(x_L^*, x_R)$. Therefore, the profile (x_L^*, x_R^*) is a PSE for $\mathcal{G}$. $\square$

Proof of Proposition 3. We prove the proposition for $1/2 < x_L^* < x_R^*$. The argument for $x_L^* < x_R^* < 1/2$ is similar. First, assume the election game $\mathcal{G}$ has a PSE with the property that $1/2 < x_L^* < x_R^*$. By Lemma 2, $x_L^* = 1/2 - \beta + \chi_L/2$ and $\chi_L + \chi_R < 4\beta$. That implies that $\frac{\chi_L}{2} > \beta > \frac{\chi_L + \chi_R}{4}$ and, therefore, that $\chi_R < \chi_L$. Using simple algebraic manipulation, it also follows that

$$\frac{\chi_L + \chi_R}{4} < \frac{\chi_L - \chi_R}{4} + \frac{\sqrt{\chi_R \cdot \chi_L}}{2} < \frac{\chi_L}{2}. \tag{3}$$

Suppose, by way of contradiction, that $2\beta < (\chi_L - \chi_R)/2 + (\chi_R \cdot \chi_L)^{1/2}$. By definition, $\pi_R(x_L^*, x_R^*) = \beta - (\chi_L - \chi_R)/2 + (\chi_L - \chi_R)^2/16\beta$. Fix any $x_R \in [1/2, x_L^*)$. Candidate R's payoff at (x_L^*, x_R) is $\pi_R(x_L^*, x_R) = (\frac{1}{2} + \frac{x_L^* + x_R - 1}{4\beta})(x_R - x_L^* + \chi_R)$. Therefore, $\lim_{x_R \to^- x_L^*} \pi_R(x_L^*, x_R) = \frac{\chi_L \chi_R}{4\beta}$. Notice that the difference between $\pi_R(x_L^*, x_R^*)$ and $\lim_{x_R \to^- x_L^*} \pi_R(x_L^*, x_R)$ gives rise to a second-order polynomial equation in β, namely, $4\beta^2 - 2\beta(\chi_L - \chi_R) + (\chi_L - \chi_R)^2/4 - \chi_L \cdot \chi_R$, which has the following two roots: $\frac{\chi_L - \chi_R}{4} \pm \frac{\sqrt{\chi_R \cdot \chi_L}}{2}$. Therefore, for any $\beta \in (\frac{\chi_L + \chi_R}{4}, \frac{\chi_L - \chi_R}{4} + \frac{\sqrt{\chi_R \cdot \chi_L}}{2})$, we have that $\pi_R(x_L^*, x_R^*) < \lim_{x_R \to^- x_L^*} \pi_R(x_L^*, x_R)$, contradicting that the strategy profile (x_L^*, x_R^*) is by hypothesis a PSE of $\mathcal{G}$. Hence, $2\beta \geqslant (\chi_L - \chi_R)/2 + (\chi_R \cdot \chi_L)^{1/2}$.

To carry out the second part of the proof, suppose $(\chi_L - \chi_R)/2 + (\chi_R \cdot \chi_L)^{1/2} \leqslant 2\beta < \chi_L$, and consider the equilibrium candidate $(x_L^*, x_R^*) = (\frac{1}{2} - \beta + \frac{\chi_L}{2}, \frac{1}{2} + \beta - \frac{\chi_R}{2})$. By the initial hypothesis and (3), we have that $\chi_L + \chi_R < 4\beta$. Therefore, since by assumption $2\beta < \chi_L$, it follows that $\chi_R < 2\beta$ and, consequently, that $1/2 < x_L^* < x_R^*$ and $p(x_L^*, x_R^*) \in (0, 1)$. Moreover, using the argument of the proof to Proposition 2, $x_L^* = \arg\max_{x_L \in [0,1]} \pi_L(x_L, x_R^*)$. To show that $x_R^* = \arg\max_{x_R \in [0,1]} \pi_R(x_L^*, x_R)$ we proceed as follows. First notice that, by applying the reasoning of the proof to Lemma 2, it can be shown that for some $\epsilon > 0$ with the property that $R_\epsilon(x_R^*) \equiv (x_R^* - \epsilon, x_R^* + \epsilon) \subset (x_L^*, \theta_R)$, $\frac{1}{2} + \beta - \frac{\chi_R}{2} = \arg\max_{x_R \in R_\epsilon(x_R^*)} \pi_R(x_L^*, x_R)$, with

$\pi_R(x_L^*, x_R^*) = \frac{\chi_R}{2} + (\beta - \frac{\chi_L}{2}) + \frac{(\chi_R-\chi_L)^2}{16\beta}$. Second, to prove that $\pi_R(x_L^*, x_R^*) > \frac{\chi_R}{2}$, observe that $\frac{\chi_R}{2} < \frac{\chi_L\chi_R}{4\beta}$ because $\chi_L/2\beta > 1$. Moreover, since $\lim_{x_R \to^- x_L^*} \pi_R(x_L^*, x_R) = \frac{\chi_L\chi_R}{4\beta}$, it also follows that $\lim_{x_R \to^- x_L^*} \pi_R(x_L^*, x_R) > \frac{\chi_R}{2}$. Thus, the desired result, i.e., $\pi_R(x_L^*, x_R^*) > \frac{\chi_R}{2}$ is obtained using the fact that, by hypothesis, $\lim_{x_R \to^- x_L^*} \pi_R(x_L^*, x_R) \leqslant \pi_R(x_L^*, x_R^*)$. The rest of the proof follows the argument of the proof to Proposition 2 and is left to the readers.[32] □

Proof of Proposition 4. Under the hypothesis of Proposition 4, i.e., $\chi_R/2 < \beta < \beta_1^C$, the existence of a MSE for the election game $\mathcal{G} = (X, \Pi_i)$ follows from the following argument. First, by Proposition 1, $\mathcal{G}$ does not possess a PSE with $x_L = x_R$ because $\chi_R < 2\beta$. Second, notice that $\beta < \beta_1^C$ implies $\chi_L/2 > \beta$ (because $\beta_1^C < \chi_L/2$). Thus, by Propositions 2 and 3, there exists no PSE with $x_L < x_R$ either. But that means, by Lemma 1, that $\mathcal{G}$ does not possess an equilibrium in pure strategies. Finally, remember that by Proposition 3 in Saporiti (2008), the mixed extension of $\mathcal{G}$ is better-reply secure; thereby $\mathcal{G}$ must admit a Nash equilibrium where at least one candidate randomizes over two or more pure strategies.

Denote by $(\mu_L^*, \mu_R^*) \in \Delta^2$ a MSE of $\mathcal{G}$, and let $\underline{x}_i$ (resp. $\bar{x}_i$) be the lower (resp. upper) bound of $\operatorname{supp}(\mu_i^*)$. That is, let $\underline{x}_i = \inf(\operatorname{supp}(\mu_i^*))$ and $\bar{x}_i = \sup(\operatorname{supp}(\mu_i^*))$, with $i = L, R$. The rest of the proof is organized in a series of claims.

Claim 1. $\operatorname{supp}(\mu_R^*) \subseteq [1/2, \theta_R]$.

Claim 1 is intuitive and follows from the fact that each location x_R smaller than $1/2$ (resp. greater than θ_R) is strictly dominated for candidate R and, therefore, it's never played with positive probability in a MSE. For the sake of brevity, the details of the proof are left for the reader, and they are available from the author upon request.

Claim 2. $\mu_L^*(\underline{x}_L) < 1$.

Proof. Suppose not. Two cases are possible. First, if $\underline{x}_L \leqslant \widetilde{x}_L(\beta, \chi_R)$, then R's best response to $\underline{x}_L$ is $x_R^* = 1/2 + \beta - \chi_R/2$. However, the profile $(\underline{x}_L, x_R^*)$ cannot be an equilibrium because under the hypothesis of Proposition 4, $\mathcal{G}$ has no equilibrium in pure strategies. Second, if $\widetilde{x}_L(\beta, \chi_R) < \underline{x}_L \leqslant \theta_R$,[33] then R's best response is to undercut L's location by choosing a position just below $\underline{x}_L$, which is not well defined because the policy space is a continuum. □

Claim 3. $\bar{x}_L \leqslant \bar{x}_R = x_R^*$.

Proof. To start, recall that a strategy profile (μ_L^*, μ_R^*) is a MSE of $\mathcal{G}$ if and only if for each candidate $i \neq j$, (1) $U_i(x, \mu_j^*) = U_i(y, \mu_j^*)$ for all $x, y \in \operatorname{supp}(\mu_i^*)$, and (2) $U_i(x, \mu_j^*) \geqslant U_i(y, \mu_j^*)$ for all $x \in \operatorname{supp}(\mu_i^*)$ and all $y \notin \operatorname{supp}(\mu_i^*)$.

To prove the first part of Claim 3, note that if $\bar{x}_L > \bar{x}_R$, then candidate L can do better by undercutting $\bar{x}_R$ from above, since for any $\epsilon > 0$ such that $\bar{x}_R < \bar{x}_L - \epsilon$

$$U_L(\bar{x}_L, \mu_R^*) = \int\limits_{X_R} \left(\frac{1}{2} + \frac{1 - x_R - \bar{x}_L}{4\beta}\right) \cdot (x_R - \bar{x}_L + \chi_L) \cdot d\mu_R^*$$
$$< \int\limits_{X_R} \left(\frac{1}{2} + \frac{1 - x_R - (\bar{x}_L - \epsilon)}{4\beta}\right) \cdot \big(x_R - (\bar{x}_L - \epsilon) + \chi_L\big) \cdot d\mu_R^* = U_L(\bar{x}_L - \epsilon, \mu_R^*).$$

To show the second part, i.e., that $\bar{x}_R = x_R^*$, consider two cases.

Case 1. Suppose $\bar{x}_L < \bar{x}_R$. On the one hand, if $\bar{x}_L \geqslant x_R^*$, then $\bar{x}_R > x_R^*$. Consider any $\epsilon > 0$ small enough such that $\bar{x}_L < \bar{x}_R - \epsilon$. Routine calculations show that

$$U_R(\mu_L^*, \bar{x}_R - \epsilon) - U_R(\mu_L^*, \bar{x}_R) = \frac{\epsilon}{4\beta} \cdot \big(2\bar{x}_R + \chi_R - 2\beta - (1 + \epsilon)\big),$$

which is strictly greater than zero because $\bar{x}_R > 1/2 + \beta - \chi_R/2 = x_R^*$, a contradiction.

On the other hand, if $\bar{x}_L < x_R^*$, then for any $x_L \in \operatorname{supp}(\mu_L^*)$, $\pi_R(x_L, \bar{x}_R) \leqslant \pi_R(x_L, x_R^*)$, with strict inequality if $\bar{x}_R \neq x_R^*$ (recall $\pi_R(x_L, \cdot)$ has a unique maximum at x_R^* above the diagonal). Integrating with respect to μ_L^*, we have that $U_R(\mu_L^*, \bar{x}_R) \leqslant U_R(\mu_L^*, x_R^*)$, with strict inequality if $\bar{x}_R \neq x_R^*$. Hence, since $\bar{x}_R \in \operatorname{supp}(\mu_R^*)$, it must be the case that $\bar{x}_R = x_R^*$.

Case 2. Suppose $\bar{x}_L = \bar{x}_R \equiv \bar{x}$. First, consider the case in which $\bar{x} < x_R^*$. For any $x_L \in [\underline{x}_L, \bar{x})$, $\pi_R(x_L, \bar{x}) < \pi_R(x_L, x_R^*)$. Integrating with respect to μ_L^* and adding $\mu_L^*(\bar{x}) \cdot \chi_R/2$ to both sides, we have

32 A complete version of it is available from the authors upon request.

33 Given that $\operatorname{supp}(\mu_R^*) \subseteq [1/2, \theta_R]$, it's never optimal for L to play above θ_R.

$$\underbrace{\int\limits_{x_L \neq \bar{x}} \pi_R(x_L, \bar{x}) \cdot d\mu_L^* + \mu_L^*(\bar{x}) \cdot \frac{\chi_R}{2}}_{=U_R(\mu_L^*, \bar{x})} < \int\limits_{x_L \neq \bar{x}} \pi_R\left(x_L, x_R^*\right) \cdot d\mu_L^* + \mu_L^*(\bar{x}) \cdot \frac{\chi_R}{2}. \tag{4}$$

Notice that $\pi_R(\bar{x}, x_R^*) = \frac{1}{\beta}(\frac{x_R^* - \bar{x}}{2} + \frac{\chi_R}{2})^2 > \frac{\chi_R}{2}$. Therefore,

$$\underbrace{\int\limits_{x_L \neq \bar{x}} \pi_R\left(x_L, x_R^*\right) \cdot d\mu_L^* + \mu_L^*(\bar{x}) \cdot \pi_R\left(\bar{x}, x_R^*\right)}_{=U_R(\mu_L^*, x_R^*)} \geqslant \int\limits_{x_L \neq \bar{x}} \pi_R\left(x_L, x_R^*\right) \cdot d\mu_L^* + \mu_L^*(\bar{x}) \cdot \frac{\chi_R}{2}, \tag{5}$$

with strict inequality if $\mu_L^*(\bar{x}) \neq 0$. Thus, combining (4) and (5), we get that $U_R(\mu_L^*, x_R^*) > U_R(\mu_L^*, \bar{x})$, contradicting that $\bar{x} \in \operatorname{supp}(\mu_R^*)$.

Second, consider the alternative case in which $\bar{x} > x_R^*$. Since μ_L^* has at most countably many atoms and X is dense in the reals, assume without loss of generality that for some $\epsilon > 0$ small enough, $\mu_L^*(\bar{x} - \epsilon) = 0$. Then,

$$\begin{aligned} U_R\left(\mu_L^*, \bar{x} - \epsilon\right) = {} & \int\limits_{\underline{x}_L}^{\bar{x}-\epsilon} \left(\frac{1}{2} + \frac{1 - x_L - (\bar{x} - \epsilon)}{4\beta}\right) \cdot (\bar{x} - \epsilon - x_L + \chi_R) \cdot d\mu_L^* \\ & + \int\limits_{\bar{x}-\epsilon}^{\bar{x}} \left(\frac{1}{2} + \frac{x_L + (\bar{x} - \epsilon) - 1}{4\beta}\right) \cdot (\bar{x} - \epsilon - x_L + \chi_R) \cdot d\mu_L^* \\ & + \mu_L^*(\bar{x}) \cdot \left(\frac{1}{2} + \frac{2\bar{x} - \epsilon - 1}{4\beta}\right) \cdot (\chi_R - \epsilon), \end{aligned} \tag{6}$$

and

$$\begin{aligned} U_R\left(\mu_L^*, \bar{x}\right) = {} & \int\limits_{\underline{x}_L}^{\bar{x}-\epsilon} \left(\frac{1}{2} + \frac{1 - x_L - \bar{x}}{4\beta}\right) \cdot (\bar{x} - x_L + \chi_R) \cdot d\mu_L^* \\ & + \int\limits_{\bar{x}-\epsilon}^{\bar{x}} \left(\frac{1}{2} + \frac{1 - x_L - \bar{x}}{4\beta}\right) \cdot (\bar{x} - x_L + \chi_R) \cdot d\mu_L^* + \mu_L^*(\bar{x}) \cdot \frac{\chi_R}{2}. \end{aligned} \tag{7}$$

Note that the difference between the first term in the right-hand side (henceforth, RHS) of the expression in (6) and the first term in the RHS of (7) is equal to

$$\frac{\epsilon}{4\beta} \cdot \left(2\bar{x} + \chi_R - 2\beta - (1 + \epsilon)\right) \cdot \int\limits_{\underline{x}_L}^{\bar{x}-\epsilon} d\mu_L^*, \tag{8}$$

which is strictly positive for $\epsilon < \bar{x} - x_R^*$ because by hypothesis $\bar{x} > x_R^*$.

Let's now consider the second term in the RHS of (6) and the second term in the RHS of (7). The difference between these two terms is equal to

$$\int\limits_{\bar{x}-\epsilon}^{\bar{x}} \underbrace{\left(\frac{x_L + \bar{x} - 1}{2\beta}\right)}_{>0} \cdot \underbrace{(\bar{x} - x_L + \chi_R)}_{>\chi_R} \cdot d\mu_L^* + \frac{\epsilon}{4\beta} \cdot (1 + \epsilon - 2\bar{x} - \chi_R - 2\beta) \cdot \int\limits_{\bar{x}-\epsilon}^{\bar{x}} d\mu_L^*. \tag{9}$$

Similarly, the difference between the last terms in the RHS of (6) and (7) is

$$\mu_L^*(\bar{x}) \cdot \left[\left(\frac{1}{2} + \underbrace{\frac{2\bar{x} - \epsilon - 1}{4\beta}}_{>0}\right) \cdot (\chi_R - \epsilon) - \frac{\chi_R}{2}\right]. \tag{10}$$

Note that (9) and (10) are both continuous in ϵ. Moreover, (9) is zero for $\epsilon = 0$, thereby it must be approximately zero for $\epsilon > 0$ arbitrarily small. In addition, the expression in (10) is strictly positive for $\epsilon = 0$ if $\mu_L^*(\bar{x}) \neq 0$ (otherwise, if $\mu_L^*(\bar{x}) = 0$, then we can just ignore these terms); and by continuity it must be nonnegative for ϵ sufficiently small. Hence, combining all this with (8), we conclude that for some $\epsilon > 0$ small enough $U_R(\mu_L^*, \bar{x} - \epsilon) > U_R(\mu_L^*, \bar{x})$, contradicting that $\bar{x} \in \operatorname{supp}(\mu_R^*)$. Therefore, $\bar{x} = x_R^*$. $\square$

Claim 4. $\underline{x}_R = \underline{x}_L \equiv \underline{x} \geqslant 1/2$.

Proof. Assume, by way of contradiction, $\underline{x}_R \neq \underline{x}_L$. On the one hand, if $\underline{x}_R < \underline{x}_L$, then by Claim 1, $1/2 < \underline{x}_L \leqslant \theta_R$, and therefore for any $\epsilon > 0$ such that $\underline{x}_R + \epsilon < \underline{x}_L$, $U_R(\mu_L^*, \underline{x}_R) < U_R(\mu_L^*, \underline{x}_R + \epsilon)$, because $\underline{x}_R + \epsilon$ raises R's probability of winning the election and, at the same time, it's closer to θ_R. But that contradicts that by definition $\underline{x}_R = \inf \operatorname{supp}(\mu_R^*)$.

On the other hand, if $\underline{x}_R > \underline{x}_L$, then we proceed as follows. Consider any $\epsilon > 0$ such that $\underline{x}_L + \epsilon < \underline{x}_R$. Routine calculations show that

$$U_L(\underline{x}_L + \epsilon, \mu_R^*) - U_L(\underline{x}_L, \mu_R^*) = \frac{\epsilon}{4\beta} \cdot (1 - \epsilon - 2\underline{x}_L + \chi_L - 2\beta); \tag{11}$$

and, since by the definition of MSE we have that $U_L(\underline{x}_L + \epsilon, \mu_R^*) \leqslant U_L(\underline{x}_L, \mu_R^*)$, it follows that $\underline{x}_L \geqslant (1 - \epsilon)/2 - \beta + \chi_L/2$ and, therefore, that $\underline{x}_R > 1/2 - \beta + \chi_L/2$, where the latter is obtained using the previous hypothesis that $\underline{x}_R > \underline{x}_L$ and an ϵ sufficiently small.

Fix any $\hat{x}_R \in \operatorname{supp}(\mu_R^*)$. For each $x_L < \hat{x}_R$, the conditional payoff function $\pi_L(x_L, \hat{x}_R) = [1/2 + (x_L + \hat{x}_R - 1)/4\beta](\hat{x}_R - x_L + \chi_L)$ has a unique maximum at $x_L^* = 1/2 - \beta + \chi_L/2$. Therefore, $\pi_L(x_L^*, \hat{x}_R) \geqslant \pi_L(\underline{x}_L, \hat{x}_R)$, with strict inequality if $\underline{x}_L \neq x_L^*$. Integrating with respect to μ_R^*, we have $U_L(x_L^*, \mu_R^*) \geqslant U_L(\underline{x}_L, \mu_R^*)$, with strict inequality if $\underline{x}_L \neq x_L^*$. Hence, it must be that $\underline{x}_L = x_L^*$.

Recall that by hypothesis $\underline{x}_R > \underline{x}_L$; and that by Claim 2 (resp. Claim 3) $\bar{x}_L > \underline{x}_L$ (resp. $x_R^* = \bar{x}_R \geqslant \bar{x}_L$). Moreover, it's easy to show that $\underline{x}_R \leqslant \bar{x}_L$.[34] Consider now an $\epsilon > 0$ such that $\underline{x}_L < \underline{x}_R - \epsilon$. Then,

$$\begin{aligned} U_L(\bar{x}_L, \mu_R^*) = &\int_{\underline{x}_R}^{\bar{x}_L} \left(\frac{1}{2} + \frac{1 - \bar{x}_L - x_R}{4\beta}\right)(x_R - \bar{x}_L + \chi_L)\, d\mu_R^* + \mu_R^*(\bar{x}_L)\frac{\chi_L}{2} \\ &+ \int_{\bar{x}_L}^{\bar{x}_R} \left(\frac{1}{2} + \frac{\bar{x}_L + x_R - 1}{4\beta}\right)(x_R - \bar{x}_L + \chi_L)\, d\mu_R^*, \end{aligned} \tag{12}$$

and

$$\begin{aligned} U_L(\underline{x}_R - \epsilon, \mu_R^*) = &\int_{\underline{x}_R}^{\bar{x}_L} \left(\frac{1}{2} + \frac{\underline{x}_R - \epsilon + x_R - 1}{4\beta}\right)\big(x_R - (\underline{x}_R - \epsilon) + \chi_L\big)\, d\mu_R^* \\ &+ \mu_R^*(\bar{x}_L)\left[\left(\frac{1}{2} + \frac{\underline{x}_R - \epsilon + \bar{x}_L - 1}{4\beta}\right)\big(\bar{x}_L - (\underline{x}_R - \epsilon) + \chi_L\big)\right] \\ &+ \int_{\bar{x}_L}^{\bar{x}_R} \left(\frac{1}{2} + \frac{\underline{x}_R - \epsilon + x_R - 1}{4\beta}\right)\big(x_R - (\underline{x}_R - \epsilon) + \chi_L\big)\, d\mu_R^*. \end{aligned} \tag{13}$$

Notice that the difference between the first terms in the RHS of (12) and (13) is negative, since for all $x_R \in [\underline{x}_R, \bar{x}_L)$ and all $\epsilon > 0$ small enough, (i) $\frac{1}{2} + \frac{1 - \bar{x}_L - x_R}{4\beta} < \frac{1}{2} + \frac{\underline{x}_R - \epsilon + x_R - 1}{4\beta}$, and (ii) $x_R - \bar{x}_L + \chi_L < x_R - (\underline{x}_R - \epsilon) + \chi_L$.

Similarly, the difference between the second terms is non-positive; that is,

$$\mu_R^*(\bar{x}_L)\left[\frac{\chi_L}{2} - \left(\frac{1}{2} + \underbrace{\frac{\underline{x}_R - \epsilon + \bar{x}_L - 1}{4\beta}}_{>0}\right)\big(\underbrace{\bar{x}_L - (\underline{x}_R - \epsilon)}_{>0} + \chi_L\big)\right] \leqslant 0,$$

with strict inequality if $\mu_R^*(\bar{x}_L) \neq 0$. Finally, the difference between the last two terms in the RHS of (12) and (13) is also smaller than or equal to zero. Indeed, for all $x_R \in (\bar{x}_L, \bar{x}_R]$, the conditional payoffs are such that $\pi_L(\bar{x}_L, x_R) \leqslant \pi_L(\underline{x}_R - \epsilon, x_R)$, since $\pi_L(\cdot, x_R)$ has a unique maximum at $x_L^* = \underline{x}_L$ and decreases above x_L^* (recall $x_L^* = \underline{x}_L < \bar{x}_R = x_R^*$ implies that $\beta > (\chi_L + \chi_R)/4$). Thus integrating with respect to μ_R^* over $(\bar{x}_L, \bar{x}_R]$ we get that $\int_{\bar{x}_L}^{\bar{x}_R} \pi_L(\bar{x}_L, x_R)\, d\mu_R^* \leqslant \int_{\bar{x}_L}^{\bar{x}_R} \pi_L(\underline{x}_R - \epsilon, x_R)\, d\mu_R^*$, as required. And combining the three previous observations, it follows that $U_L(\bar{x}_L, \mu_R^*) < U_L(\underline{x}_R - \epsilon, \mu_R^*)$, a contradiction. Hence, $\underline{x}_R = \underline{x}_L \equiv \underline{x}$; and by Claim 1, $\underline{x} \geqslant 1/2$. $\square$

[34] Otherwise, for any $\hat{x}_R \in \operatorname{supp}(\mu_R^*)$, $\pi_L(\underline{x}_L, \hat{x}_R) > \pi_L(\bar{x}_L, \hat{x}_R)$, and integrating with respect to μ_R^* we would find the desired contradiction, i.e., $U_L(\underline{x}_L, \mu_R^*) > U_L(\bar{x}_L, \mu_R^*)$.

Claim 5. $\underline{x} = \widetilde{x}_L(\beta, \chi_R)$.

Proof. By Claims 1–4, $\operatorname{supp}(\mu_L^*) \subseteq [1/2, x_R^*]$ and $\bar{x}_R > \underline{x}$; hence, $\mu_R^*(\underline{x}) < 1$. Assume, by contradiction, $\underline{x} > \widetilde{x}_L(\beta, \chi_R)$. (The other case is similar.) By the definition of MSE, for any $\epsilon > 0$ small enough, $U_R(\mu_L^*, x_R^*) \geqslant U_R(\mu_L^*, \underline{x} - \epsilon)$, where

$$U_R(\mu_L^*, x_R^*) = \mu_L^*(\underline{x}) \cdot \pi_R(\underline{x}, x_R^*) + \int\limits_{x_L \neq \underline{x}} \left(\frac{1}{2} + \frac{1 - x_L - x_R^*}{4\beta}\right) \cdot (x_R^* - x_L + \chi_R) \cdot d\mu_L^* + \mu_L^*(x_R^*) \frac{\chi_R}{2}, \tag{14}$$

and

$$U_R(\mu_L^*, \underline{x} - \epsilon) = \mu_L^*(\underline{x}) \cdot \pi_R(\underline{x}, \underline{x} - \epsilon) + \int\limits_{x_L \neq \underline{x}} \left(\frac{1}{2} + \frac{\underline{x} - \epsilon + x_L - 1}{4\beta}\right) \cdot (\underline{x} - \epsilon - x_L + \chi_R) \cdot d\mu_L^*. \tag{15}$$

Note that since by hypothesis $\underline{x} > \widetilde{x}_L(\beta, \chi_R)$, we have that $\limsup_{x_R \to^- \underline{x}} \pi_R(\underline{x}, x_R) > \pi_R(\underline{x}, x_R^*)$. Therefore,

$$\mu_L^*(\underline{x}) \cdot \left[\pi_R(\underline{x}, \underline{x} - \epsilon) - \pi_R(\underline{x}, x_R^*)\right] > 0. \tag{16}$$

Applying once again the definition of a mixed strategy equilibrium, Claims 3 and 4 imply that $U_R(\mu_L^*, x_R^*) = U_R(\mu_L^*, \underline{x})$. Thus,

$$\int\limits_{x_L \neq \underline{x}} \pi_R(x_L, \underline{x})\, d\mu_L^* = \mu_L^*(\underline{x}) \cdot \left[\pi_R(\underline{x}, x_R^*) - \frac{\chi_R}{2}\right] + \int\limits_{x_L \neq \underline{x}} \pi_R(x_L, x_R^*)\, d\mu_L^* + \mu_L^*(x_R^*) \frac{\chi_R}{2}. \tag{17}$$

If $\underline{x} < \frac{1}{2} + \beta - \frac{\chi_R}{2} + (\chi_R - \sqrt{2\beta\chi_R})$, then $\pi_R(\underline{x}, x_R^*) > \frac{\chi_R}{2}$. Hence, (17) implies that

$$\int\limits_{x_L \neq \underline{x}} \pi_R(x_L, \underline{x})\, d\mu_L^* > \int\limits_{x_L \neq \underline{x}} \pi_R(x_L, x_R^*)\, d\mu_L^* + \mu_L^*(x_R^*) \frac{\chi_R}{2}. \tag{18}$$

Notice that the left-hand side of (18) is left continuous in x_R at $\underline{x}$, since $\pi_R(x_L, \underline{x}) = (\frac{1}{2} + \frac{x_L + \underline{x} - 1}{4\beta}) \cdot (\underline{x} - x_L + \chi_R)$, meaning that for $\epsilon > 0$ sufficiently small,

$$\int\limits_{x_L \neq \underline{x}} \pi_R(x_L, \underline{x} - \epsilon)\, d\mu_L^* \geqslant \int\limits_{x_L \neq \underline{x}} \pi_R(x_L, x_R^*)\, d\mu_L^* + \mu_L^*(x_R^*) \frac{\chi_R}{2}. \tag{19}$$

Thus, combining (16) and (19), it follows from (14) and (15) that $U_R(\mu_L^*, x_R^*) < U_R(\mu_L^*, \underline{x} - \epsilon)$, a contradiction.

Alternatively, if $\underline{x} \geqslant \frac{1}{2} + \beta - \frac{\chi_R}{2} + (\chi_R - \sqrt{2\beta\chi_R})$, then

$$\pi_R(\underline{x}, x_R^*) \leqslant \frac{\chi_R}{2}; \tag{20}$$

and from (17) we have that

$$\int\limits_{x_L \neq \underline{x}} \pi_R(x_L, \underline{x})\, d\mu_L^* \leqslant \int\limits_{x_L \neq \underline{x}} \pi_R(x_L, x_R^*)\, d\mu_L^* + \mu_L^*(x_R^*) \frac{\chi_R}{2}. \tag{21}$$

Using again the continuity of $\pi_R(x_L, x_R) = (\frac{1}{2} + \frac{x_L + x_R - 1}{4\beta}) \cdot (x_R - x_L + \chi_R)$ in x_R at $\underline{x}$, for $\epsilon > 0$ small enough

$$\int\limits_{x_L \neq \underline{x}} \pi_R(x_L, \underline{x} - \epsilon)\, d\mu_L^* \approx \int\limits_{x_L \neq \underline{x}} \pi_R(x_L, \underline{x})\, d\mu_L^*. \tag{22}$$

By definition, $\widetilde{x}_L(\beta, \chi_R) \equiv 1/2 \cdot (1 + 2\beta + 3\chi_R - 2\sqrt{2}\sqrt{2\beta\chi_R + \chi_R^2})$. Thus, since by the hypothesis of Proposition 4 $\chi_R < 2\beta$, we have that $\widetilde{x}_L(\beta, \chi_R) > 1/2$, which implies that $\underline{x} > 1/2$ as well (recall we assumed before $\underline{x} > \widetilde{x}_L$). Hence, by the discontinuity of $p(\cdot)$ at $(\underline{x}, \underline{x})$, $p(\underline{x}, \underline{x} - \epsilon)$ is well above $1/2$, meaning that for $\epsilon > 0$ sufficiently close to zero

$$\pi_R(\underline{x}, \underline{x} - \epsilon) = \left(\frac{1}{2} + \frac{2\underline{x} - (1 + \epsilon)}{4\beta}\right)(\chi_R - \epsilon) > \frac{\chi_R}{2}. \tag{23}$$

Finally, from (17),

$$\mu_L^*(\underline{x}) \cdot \left[\pi_R(\underline{x}, x_R^*) - \frac{\chi_R}{2}\right] + \int\limits_{x_L \neq \underline{x}} \left[\pi_R(x_L, x_R^*) - \pi_R(x_L, \underline{x})\right] d\mu_L^* + \mu_L^*(x_R^*) \frac{\chi_R}{2} = 0; \tag{24}$$

and combining (20), (22) and (23) and comparing them with (24), the expression below

$$\mu_L^*(\underline{x})\cdot\left[\pi_R\left(\underline{x},x_R^*\right)-\pi_R(\underline{x},\underline{x}-\epsilon)\right]+\int\limits_{x_L\neq\underline{x}}\left[\pi_R\left(x_L,x_R^*\right)-\pi_R(x_L,\underline{x}-\epsilon)\right]d\mu_L^*+\mu_L^*\left(x_R^*\right)\frac{\chi_R}{2}\tag{25}$$

turns out to be strictly smaller than zero. However, that means that $U_R(\mu_L^*,x_R^*)<U_R(\mu_L^*,\underline{x}-\epsilon)$, contradicting that $x_R^*\in\operatorname{supp}(\mu_R^*)$. Therefore, $\underline{x}=\widetilde{x}_L(\beta,\chi_R)$. $\square$

Claim 6. *If $\beta\leqslant\frac{\chi_L+\chi_R}{4}$, then $\bar{x}_L=x_R^*$.*

Proof. Suppose, by way of contradiction, that $\bar{x}_L<x_R^*$. (Recall that by Claim 3, $\bar{x}_L\leqslant x_R^*$.) Then, for any $x',x''\in(\bar{x}_L,x_R^*)$, with $x'<x''$, we have that $\pi_R(x_L,x'')>\pi_R(x_L,x')$ for all $x_L\in\operatorname{supp}(\mu_L^*)$, because $\pi_R(x_L,\cdot)$ is strictly increasing on $(\bar{x}_L,x_R^*)$.[35] Integrating with respect to x_L over $\operatorname{supp}(\mu_L^*)$, we get that $U_R(\mu_L^*,x'')>U_R(\mu_L^*,x')$; and since this holds for any $x'<x''$, it follows that (i) R does not allocate probability mass on $(\bar{x}_L,x_R^*)$, and (ii) by Claim 3, μ_R^* has an atom at x_R^*, i.e., $\mu_R^*(x_R^*)>0$. The rest of the proof shows that candidate L would profitably undercut x_R^* from below.

To do that, first we prove that $\mu_R^*(\bar{x}_L)=0$. That follows by considering the difference between the left-wing candidate's conditional expected payoff at $\bar{x}_L$ and at $\bar{x}_L-\epsilon$, with $\epsilon>0$ arbitrarily small, which is equal to

$$\begin{aligned}U_L\left(\bar{x}_L,\mu_R^*\right)-U_L\left(\bar{x}_L-\epsilon,\mu_R^*\right)={}&\int\limits_{\underline{x}}^{\bar{x}_L-\epsilon}\left[\pi_L(\bar{x}_L,x_R)-\pi_L(\bar{x}_L-\epsilon,x_R)\right]d\mu_R^*\\&+\int\limits_{\bar{x}-\epsilon}^{\bar{x}_L}\left[\pi_L(\bar{x}_L,x_R)-\pi_L(\bar{x}_L-\epsilon,x_R)\right]d\mu_R^*\\&+\mu_R^*\left(x_R^*\right)\left[\pi_L\left(\bar{x}_L,x_R^*\right)-\pi_L\left(\bar{x}_L-\epsilon,x_R^*\right)\right]\\&+\mu_R^*(\bar{x}_L)\left[\frac{\chi_L}{2}-\pi_L(\bar{x}_L-\epsilon,\bar{x}_L)\right].\end{aligned}\tag{26}$$

Using the continuity of the payoff function outside the main diagonal and the fact that ϵ is by hypothesis arbitrarily small, the first three terms of the RHS of (26) are arbitrarily close to zero. Therefore, since $\frac{\chi_L}{2}<\pi_L(\bar{x}_L-\epsilon,\bar{x}_L)$, the fact that $\bar{x}_L\in\operatorname{supp}(\mu_L^*)$ implies that $\mu_R^*(\bar{x}_L)=0$. (Otherwise, we would have that $U_L(\bar{x}_L,\mu_R^*)<U_L(\bar{x}_L-\epsilon,\mu_R^*)$, which would contradict that (μ_L^*,μ_R^*) is by hypothesis a MSE of $\mathcal{G}$.)

Second, we work out candidate R's probability mass on x_R^* by equalizing the left-wing candidate's conditional expected payoffs at $\underline{x}$ and $\bar{x}_L$, which turns out to be

$$\mu_R^*\left(x_R^*\right)=\frac{\mu_R^*(\underline{x})[\frac{\chi_L}{2}-\pi_L(\bar{x}_L,\underline{x})]+\int_{\underline{x}}^{\bar{x}_L}[\pi_L(\underline{x},x_R)-\pi_L(\bar{x}_L,x_R)]\,d\mu_R^*}{\pi_L(\bar{x}_L,x_R^*)-\pi_L(\underline{x},x_R^*)}.\tag{27}$$

Finally, notice that

$$\begin{aligned}U_L\left(x_R^*-\epsilon,\mu_R^*\right)-U_L\left(\bar{x}_L,\mu_R^*\right)={}&\int\limits_{\underline{x}}^{\bar{x}_L}\underbrace{\left[\pi_L\left(x_R^*-\epsilon,x_R\right)-\pi_L(\bar{x}_L,x_R)\right]}_{<0\ \forall x_R\in(\underline{x},\bar{x}_L)}d\mu_R^*\\&+\mu_R^*\left(x_R^*\right)\underbrace{\left[\pi_L\left(x_R^*-\epsilon,x_R^*\right)-\pi_L\left(\bar{x}_L,x_R^*\right)\right]}_{>0\text{ because }\pi_L'(\cdot,x_R^*)>0},\end{aligned}\tag{28}$$

and replacing (27) into (28), we get the desired contradiction, namely, $U_L(x_R^*-\epsilon,\mu_R^*)>U_L(\bar{x}_L,\mu_R^*)$. Therefore, $\bar{x}_L=x_R^*$. $\square$

Claim 7. *If $\beta>\frac{\chi_L+\chi_R}{4}$, then $\bar{x}_L=x_L^*<x_R^*$.*

Proof. The claim is proved following the same type of reasoning we have applied before in the proof of Claim 6. (The fact that $x_L^*<x_R^*$ is shown in the proof of Lemma 2.) The only main difference is that the second term in the RHS of (28) is not anymore positive when $\beta>\frac{\chi_L+\chi_R}{4}$, because the conditional payoff function $\pi_L(\cdot,x_R^*)$ is decreasing above x_L^*. That explains why undercutting the right-wing candidate's upper bound policy x_R^* is not anymore profitable for candidate L. $\square$

[35] In fact, $\pi_R(x_L,\cdot)$ is strictly concave with a maximum at x_R^*.

Claim 8. *If $\beta \leqslant \frac{\chi_L+\chi_R}{4}$, then $\text{supp}(\mu_i^*) = [\underline{x}, \bar{x}]$ for all $i = L, R$, with $\underline{x} = \widetilde{x}_L(\beta, \chi_R)$ and $\bar{x} = \frac{1}{2} + \beta - \frac{\chi_R}{2} = x_R^*$.*

Proof. The fact that for all i, $\underline{x}_i = \widetilde{x}_L(\beta, \chi_R)$ (respectively, $\bar{x}_i = x_R^*$) follows from Claim 5 (respectively, from Claims 3 and 6). Thus, it remains to be shown that $\text{supp}(\mu_i^*)$ is an interval. Without loss of generality, consider $x \in (\underline{x}, \bar{x})$ and assume, by way of contradiction, that $x \notin \text{supp}(\mu_R^*)$. The other case, i.e., $x \notin \text{supp}(\mu_L^*)$, is analogous.

By definition of $\text{supp}(\mu_R^*)$, there exists $\epsilon > 0$ such that $\mu_R^*([x-\epsilon, x+\epsilon] \cap X) = 0$. Consider any two alternatives $x', x'' \in [x-\epsilon, x+\epsilon]$, with $x' < x''$. Since $\pi_L(\cdot, x_R)$ is increasing for all $x_R \in (x+\epsilon, x_R^*]$, it is easy to show that $U_L(x'', \mu_R^*) > U_L(x', \mu_R^*)$. Therefore, $x' \notin \text{supp}(\mu_L^*)$; and repeating the argument, it follows that μ_L^* has an atom at $x+\epsilon$. But then R must find it profitable to undercut $x+\epsilon$ from below (recall $x+\epsilon > \widetilde{x}_L$), contradicting that by hypothesis $\mu_R^*([x-\epsilon, x+\epsilon] \cap X) = 0$. □

Claim 9. *If $\beta > \frac{\chi_L+\chi_R}{4}$, then $\text{supp}(\mu_L^*) = [\underline{x}, \bar{x}]$ and $\text{supp}(\mu_R^*) = [\underline{x}, \bar{x}] \cup \{x_R^*\}$, with $\underline{x} = \widetilde{x}_L(\beta, \chi_R)$ and $\bar{x} = \frac{1}{2} - \beta + \frac{\chi_L}{2} = x_L^*$.*

Proof. The fact that $\bar{x}_L = x_L^*$ follows from Claim 7. To show that $\mu_R^*((x_L^*, x_R^*)) = 0$, we use the argument of the proof of Claim 6. To be more precise, consider any $x', x'' \in (\bar{x}_L, x_R^*)$, with $x' < x''$. Since for all $x_L \in [\underline{x}, \bar{x}]$, the conditional payoff $\pi_R(x_L, \cdot)$ is strictly increasing on $(\bar{x}_L, x_R^*)$, we have that $\pi_R(x_L, x'') > \pi_R(x_L, x')$. Integrating with respect to x_L over $\text{supp}(\mu_L^*)$, we get that $U_R(\mu_L^*, x'') > U_R(\mu_L^*, x')$. Hence, since the pair $x' < x''$ was arbitrarily chosen, it follows that candidate R does not allocate probability mass on $(\bar{x}_L, x_R^*)$. The rest of the proof is similar to the proof of Claim 8. □

To consider the analogous characterization for the case where the left-wing candidate is the relatively more ideological candidate (see Fig. 2(b)), define $\widetilde{x}_R(\beta, \chi_L)$ as the solution to $\Pi_L(x_L^*, x_R') - \limsup_{x_L \to^+ x_R'} \Pi_L(x_L, x_R') = 0$. Then:

Proposition 5 *(Probabilistic differentiation). If $\chi_L/2 < \beta < \beta_2^C$, the election game $\mathcal{G} = (X, \Pi_i)_{i=L,R}$ has a mixed strategy equilibrium $(\mu_L^*, \mu_R^*) \in \Delta^2$ with the property that,*

(a) *If $\beta \leqslant \frac{\chi_L+\chi_R}{4}$, then $\text{supp}(\mu_i^*) = [\underline{x}, \bar{x}]$ for all $i = L, R$, with $\underline{x} = \frac{1}{2} - \beta + \frac{\chi_L}{2} = x_L^*$ and $\bar{x} = \widetilde{x}_R(\beta, \chi_L)$; and*
(b) *If $\beta > \frac{\chi_L+\chi_R}{4}$, then $\text{supp}(\mu_R^*) = [\underline{x}, \bar{x}]$ and $\text{supp}(\mu_L^*) = [\underline{x}, \bar{x}] \cup \{x_L^*\}$, with $\underline{x} = \frac{1}{2} + \beta - \frac{\chi_R}{2} = x_R^*$ and $\bar{x} = \widetilde{x}_R(\beta, \chi_L)$.*

Proof of Proposition 5. Analogous to the proof of Proposition 4. □

Appendix B. Supplementary material

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.geb.2013.10.004.

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